TEKS Cluster: Linear Functions

A.2 **Linear functions, equations, and inequalities.** The student applies the mathematical process standards when using properties of linear functions to write and represent in multiple ways, with and without technology, linear equations, inequalities, and systems of equations.

A.3 **Linear functions, equations, and inequalities.** The student applies the mathematical process standards when using graphs of linear functions, key features, and related transformations to represent in multiple ways and solve, with and without technology, equations, inequalities, and systems of equations.

Connected Knowledge and Skills A.4, A.5, A.12

Solving Linear Equations

Readiness Standards

A.5(A) solve linear equations in one variable, including those for which the application of the distributive property is necessary and for which variables are included on both sides

Supporting Standards

A.2(D) write and solve equations involving direct variation

A.12(E) solve mathematic and scientific formulas, and other literal equations, for a specified variable

Writing Linear Equations

Readiness Standards

A.2(C) write linear equations in two variables given a table of values, a graph, and a verbal description

A.3(B) calculate the rate of change of a linear function represented tabularly, graphically, or algebraically in context of mathematical and real-world problems

Supporting Standards

A.2(B) write linear equations in two variables in various forms, including $y = mx + b$, $Ax + By = C$, and $y - y_1 = m(x - x_1)$, given one point and the slope and given two points

A.2(E) write the equation of a line that contains a given point and is parallel to a given line

A.2(F) write the equation of a line that contains a given point and is perpendicular to a given line

A.2(G) write an equation of a line that is parallel or perpendicular to the *x*- or *y*-axis and determine whether the slope of the line is zero or undefined

A.3(A) determine the slope of a line given a table of values, a graph, two points on the line, and an equation written in various forms, including $y = mx + b$, $Ax + By = C$, and $y - y_1 = m(x - x_1)$

A.4(C) write, with and without technology, linear functions that provide a reasonable fit to data to estimate solutions and make predictions for real-world problems

A.12(C) identify terms of arithmetic and geometric sequences when the sequences are given in function form using recursive processes

A.12(D) write a formula for the n^{th} term of arithmetic and geometric sequences, given the value of several of their terms

Describing Linear Functions

Readiness Standards

A.2(A) determine the domain and range of a linear function in mathematical problems; determine reasonable domain and range values for real-world situations, both continuous and discrete; and represent domain and range using inequalities

A.3(C) graph linear functions on the coordinate plane and identify key features, including *x*-intercept, *y*-intercept, zeros, and slope, in mathematical and real-world problems

Supporting Standards

A.3(E) determine the effects on the graph of the parent function $f(x) = x$ when $f(x)$ is replaced by $af(x)$, $f(x) + d$, $f(x - c)$, $f(bx)$ for specific values of *a*, *b*, *c*, and *d*

A.4(A) calculate, using technology, the correlation coefficient between two quantitative variables and interpret this quantity as a measure of the strength of the linear association

A.4(B) compare and contrast association and causation in real-world problems

A.12(A) decide whether relations represented verbally, tabularly, graphically, and symbolically define a function

A.12(B) evaluate functions, expressed in function notation, given one or more elements in their domains

TEKS Scaffold

TEKS	Student Expectation
2A.3(B)	solve systems of three linear equations in three variables by using Gaussian elimination, technology with matrices, and substitution (R)
A.5(A)	**A.5 Linear functions, equations, and inequalities.** The student applies the mathematical process standards to solve, with and without technology, linear equations and evaluate the reasonableness of their solutions. The student is expected to: **(A) solve linear equations in one variable, including those for which the application of the distributive property is necessary and for which variables are included on both sides**
8.8(C)	model and solve one-variable equations with variables on both sides of the equal sign that represent mathematical and real-world problems using rational number coefficients and constants (R)
7.11(A)	model and solve one-variable, two-step equations and inequalities (R)
6.10(A)	model and solve one-variable, one-step equations and inequalities that represent problems, including geometric concepts (R)

Stimulus

Word Problem	Verbal Description	Chart/Table	Graph
Equation/ Expression*	Manipulatives	Diagram/Image	Number Line
Base Ten Blocks	Measurement Tool	Formula	Geometric Figures

Content Builder (see Appendix for Tree Diagram)

- Solve linear equations in one variable for which the application of the distributive property is necessary
- Solve linear equations in one variable for which variables are included on both sides

Instructional Implications

Students should solve various types of linear, one-variable equations. Instruction can begin with a review of easier equation-solving strategies learned in middle school (one-step and two-step equations in one variable) and should include a discussion of the properties of equality (adding the same value to both sides of an equation or dividing both sides of an equation by the same number, etc.).

In Algebra I, students are expected to solve multi-step equations that require the use of the distributive property and combining like terms.

For example, consider the following equation: $5(2x - 4) - 3(x - 2) = 4x - 2$

Students must simplify the expression on the left-hand side of the equation before using the properties of equality to solve. This involves using the distributive property and combining like terms.

$$5(2x-4)-3(x-2)=4x-2$$
$$10x-20-3x+6=4x-2$$
$$10x-3x-20+6=4x-2$$
$$7x-14=4x-2$$

Students then move the variables to the same side of the equal sign by adding the opposite of one of these variable terms to both sides of the equation (sometimes referred to as "cancelling" variables from one side).

$$\begin{aligned} 7x-14&=4x-2 \\ -4x\quad &\quad -4x \\ 3x-14&=-2 \end{aligned}$$

The last steps in the equation-solving process are to isolate the variable by adding or subtracting constants on both sides and dividing both sides by the coefficient of the variable.

$$\begin{aligned} 3x-14&=-2 \\ +14\quad &\quad +14 \\ \frac{3x}{3}&=\frac{12}{3} \\ x&=4 \end{aligned}$$

Learning from Mistakes

Students may make the following mistakes:
- Making arithmetic errors with integers when combining like terms*
- When applying the distributive property, not distributing a coefficient to both terms inside the parentheses*
- Being inconsistent in their attempt to keep the equation balanced (e.g., adding 14 to one side of the equation but not the other or attempting to add 14 to each term in the equation)*
- Failing to distribute a negative sign when a subtraction symbol is in front of terms grouped with parentheses*

Academic Vocabulary

distributive property

Interesting Items

A.5(A) 2017 #11
A.5(A) 2016 #8

A.2(D) **A.2 Linear functions, equations, and inequalities.** The student applies the mathematical process standards when using properties of linear functions to write and represent in multiple ways, with and without technology, linear equations, inequalities, and systems of equations. The student is expected to:

(D) write and solve equations involving direct variation

Stimulus

Word Problem*	Verbal Description*	Chart/Table	Graph
Equation/ Expression	Manipulatives	Diagram/Image	Number Line
Base Ten Blocks	Measurement Tool	Formula	Geometric Figures

Academic Vocabulary

constant of proportionality
direct variation
directly proportional*
function*
varies directly*

Interesting Items

A.2(D) 2021 #42
A.2(D) 2016 #42

Role in Concept Development

Supports

- A.2(C) write linear equations in two variables given a table of values, a graph, and a verbal description
- A.3(B) calculate the rate of change of a linear function represented tabularly, graphically, or algebraically in context of mathematical and real-world problems

Connection/ Relevance

Direct variation covers the special case of linear functions when the slope is y/x, which can be contrasted with the general slope formula. Direct variation also deals with the special case of linear functions when the y-intercept is (0,0), which can be contrasted with the general $y = mx + b$ equation.

When to Teach

Before/Prerequisite to A.2(C) and A.3(B)

Instructional Implications

Instruction should include writing and solving equations that involve direct variation. Direct variation equations are relationships of the form $y = kx$, where k is called the constant of proportionality. Students are expected to determine the constant of proportionality from a pair of x- and y-values using $k = \frac{y}{x}$, then write the equation in $y = kx$ form. From this equation, students can generate other ordered pairs of x- and y-values. For example:

Item	Solution
If y varies directly as x and $y = 135$ when $x = 90$, what is the value of y when $x = 24$?	$y = kx$ $135 = k(90)$ $k = \frac{135}{90} = 1.5$ $y = 1.5x$ when $x = 24$, $y = 1.5(24) = 36$.

Learning from Mistakes

Students may make the following mistakes:

- Substituting incorrectly into the equation $y = kx$
- Solving for k but forgetting to solve for additional values of x or y*
- Over-generalizing that all linear relationships involve direct variation

A.12(E) Supporting

A.12(E)

A.12 Number and algebraic methods. The student applies the mathematical process standards and algebraic methods to write, solve, analyze, and evaluate equations, relations, and functions. The student is expected to:

(E) solve mathematic and scientific formulas, and other literal equations, for a specified variable

Stimulus

Word Problem	Verbal Description	Chart/Table	Graph
Equation/ Expression*	Manipulatives	Diagram/Image	Number Line
Base Ten Blocks	Measurement Tool	Formula	Geometric Figures

Academic Vocabulary

equivalent*
literal equation

Interesting Items

A.12(E) 2016 #18

Role in Concept Development

Supports

- A.5(A) solve linear equations in one variable, including those for which the application of the distributive property is necessary and for which variables are included on both sides.
- A.8(A) solve quadratic equations having real solutions by factoring, taking square roots, completing the square, and applying the quadratic formula

Connection/ Relevance

Students should be able to solve linear and quadratic equations for a specific variable within a variety of formulas.

When to Teach

With A.5(A) and A.8(A)

Instructional Implications

Instruction should include solving literal equations for a specified variable. Literal equations, such as formulas, contain multiple variables. The process of solving literal equations involves isolating a specified variable by adding, subtracting, multiplying, and dividing by values on both sides of an equation and can include taking square roots. For example, consider the standard form of a linear equation, $Ax + By = C$. Students may be expected to solve this literal equation for y (the steps are shown below).

$$Ax + By = C$$
$$-Ax \qquad -Ax$$
$$\frac{By}{B} = \frac{C - Ax}{B}$$

- Start with the standard form of a linear equation.
- Subtract the Ax term from both sides.
- Divide both sides by B.

$$y = \frac{C - Ax}{B}$$

- The answer can be written as shown, or by distributing the division by B, a student could also write $y = \frac{C}{B} - \frac{A}{B}x$.

For examples of literal equations, students should use other mathematical or scientific formulas.

Formula	Solve	
Force = (mass)(acceleration) $F = ma$	Solve for m: $\frac{F}{a} = m$	Solve for a: $\frac{F}{m} = a$
The formula for the volume of a cylinder $V = \pi r^2 h$	Solve for h: $\frac{V}{\pi r^2} = h$	Solve for r: $\sqrt{\frac{V}{\pi h}} = r$

Learning from Mistakes

Students may make the following mistakes:

- Combining unlike terms*
- Incorrectly canceling terms

TEKS Scaffold

TEKS	Student Expectation
A.2(C)	**A.2 Linear functions, equations, and inequalities.** The student applies the mathematical process standards when using properties of linear functions to write and represent in multiple ways, with and without technology, linear equations, inequalities, and systems of equations. The student is expected to: **(C) write linear equations in two variables given a table of values, a graph, and a verbal description**
8.5(I)	write an equation in the form $y = mx + b$ to model a linear relationship between two quantities using verbal, numerical, tabular, and graphical representations (R)
7.7(A)	represent linear relationships using verbal descriptions, tables, graphs, and equations that simplify to the form $y = mx + b$ (R)

Stimulus

Word Problem*	Verbal Description	Chart/Table*	Graph*
Equation/ Expression*	Manipulatives	Diagram/Image	Number Line
Base Ten Blocks	Measurement Tool	Formula	Geometric Figures

Academic Vocabulary

linear equation*
slope*
y-intercept*

Interesting Items

A.2(C) 2018 #43
A.2(C) 2017 #50
A.2(C) 2016 #35

Content Builder (see Appendix for Tree Diagram)

- Write linear equations in two variables given a table of values
- Write linear equations in two variables given a graph
- Write linear equations in two variables given a verbal description

Instructional Implications

Students should write linear equations in two variables from a variety of prompts. Students must use a table of values to generate a linear equation. The following examples demonstrate ways students can derive an equation from a table of values:

x	y
2	8
4	6
9	1

- Students may recognize a relationship between the values in a table. In this example, a student may recognize that the *x*- and *y*-values add up to 10. The linear equation for the table would be $x + y = 10$.

x	y
0	5
1	8
2	11
3	14

- Students can determine the equation from patterns in the values in a table. Here, the *y*-values start on 5 and increase each time by 3. Since the *x*-values are sequential (0, 1, 2, ...), students can derive the equation $y = 5 + 3x$ (or $y = 3x + 5$).

x	y
-2	5
6	1

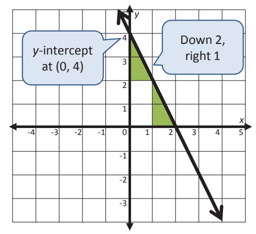
- Students must rely on the formula for determining the slope of the line, then use the point-slope form to write a linear equation. Here the slope is $m = \frac{1-5}{6-(-2)} = \frac{-4}{8} = -\frac{1}{2}$. Then, the equation can be written using the point-slope form: $y - 1 = -\frac{1}{2}(x - 6)$.

(continued)

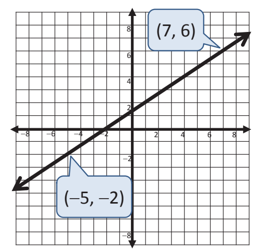
 * Used on STAAR

Instructional Implications (continued)

Students must also be able to write the equation of a line from the graph.



- Students may be able to write the equation in slope-intercept form by identifying the slope and *y*-intercept directly from the graph. In this example, the *y*-intercept is (0, 4), and the slope is -2. The equation for the line can be written as $y = -2x + 4$.



- The slope and *y*-intercept may not be easy to identify directly from the graph. In this case, students can begin by identifying two ordered pairs on the line, then utilize the formulas for slope and the point-slope form of a linear equation.

  Here the slope is $m = \frac{6 - (-2)}{7 - (-5)} = \frac{8}{12} = \frac{2}{3}$.

  The equation can be written using the point-slope form: $y - 6 = \frac{2}{3}(x - 7)$.

Students must also be able to write the equation of a line from a verbal description. It is important for students to define variables and to associate slope with a rate of change, unit rate, or an amount of gradual increase. Likewise, students should relate a starting point, initial amount, or flat fee with the *y*-intercept of a linear equation.

Verbal description	A plumber's total charge is determined by a $40 per hour fee, but he also charges $60 to come out to your home.	As we drive, our distance from home changes over time. Right now, we are 440 miles from home, but we are driving back at a speed of 60 mph.
Variables	*h* = time in hours *c* = total charge	*t* = time in hours *d* = distance (miles) from home
y-intercept	60 (the initial fee or flat rate)	440 (the current location)
Slope	40 (because the charge will increase $40 each hour)	-60 (because the distance from home is decreasing at 60 mph)
Equation	$c = 40h + 60$	$d = -60 + 440$

Learning from Mistakes

Students may make the following mistakes:
- Confusing the *y*-intercept with the *x*-intercept*
- Switching values for *x* and *y* in the slope formula, or in the point-slope form of a linear equation*
- Confusing the signs of a line's slope or *y*-intercept (positive or negative)
- Having difficulty representing equations in different forms*

TEKS Scaffold

TEKS	Student Expectation
A.3(B)	**A.3 Linear functions, equations, and inequalities.** The student applies the mathematical process standards when using graphs of linear functions, key features, and related transformations to represent in multiple ways and solve, with and without technology, equations, inequalities, and systems of equations. The student is expected to: **(B) calculate the rate of change of a linear function represented tabularly, graphically, or algebraically in context of mathematical and real-world problems**
8.4(C)	use data from a table or graph to determine the rate of change or slope and *y*-intercept in mathematical and real-world problems (R)
7.4(A)	represent constant rates of change in mathematical and real-world problems given pictorial, tabular, verbal, numeric, graphical, and algebraic representations, including *d* = *rt* (R)

Stimulus

Word Problem*	Verbal Description	Chart/Table*	Graph*
Equation/ Expression*	Manipulatives	Diagram/Image	Number Line
Base Ten Blocks	Measurement Tool	Formula	Geometric Figures

Content Builder (see Appendix for Tree Diagram)

- Calculate the rate of change of a linear function represented:
  - tabularly in a mathematical context
  - tabularly in a real-world context
  - graphically in a mathematical context
  - graphically in a real-world context
  - algebraically in a mathematical context
  - algebraically in a real-world context

Instructional Implications

Students should calculate the rate of change of a linear function from a variety of representations and contexts. Instruction should include determining the rate of change from a table of values. In real-world contexts, this can be introduced by asking questions such as, "How fast is this quantity changing?"

Time (hours)	Height (inches)
0	10
2	7
5	2.5

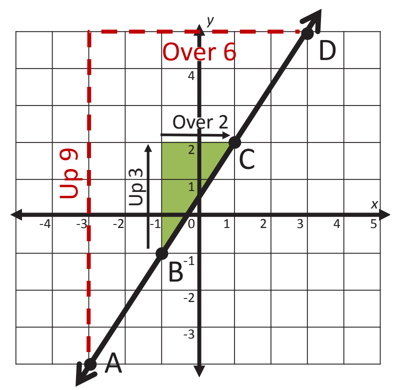
The table at left charts the height of a candle as it burns over time. Over the first 2 hours, the candle's height decreased 3 inches. The rate of change would be (-3 in)/(2 hr) = -1.5 in/hr.

Note that the rate of change remains constant for any given linear function. In the next 3-hour period (from 2 hr to 5 hr), the candle's height decreased 4.5 inches (since 2.5 – 7 = –4.5). The rate of change would be (-4.5 in)/(3 hr), which is still -1.5 in/hr.

From such examples, students can develop the formulas for rate of change related to slope:

$$\text{Rate of change} = \text{slope} = m = \frac{\text{Change in } y\text{-values}}{\text{Change in } x\text{-values}} = \frac{y_2 - y_1}{x_2 - x_1}$$

Instruction should also include determining rate of change (slope) from graphical representations. In mathematical contexts, this process can be simplified to counting the change in *y* (up or down, "rise") and the change in *x* (to the right, "run") along the grid lines from point to point on a line's graph.



In the example shown at left, point C is 3 units up and 2 units over to the right from point B. The rate of change or slope is $\frac{3}{2}$.

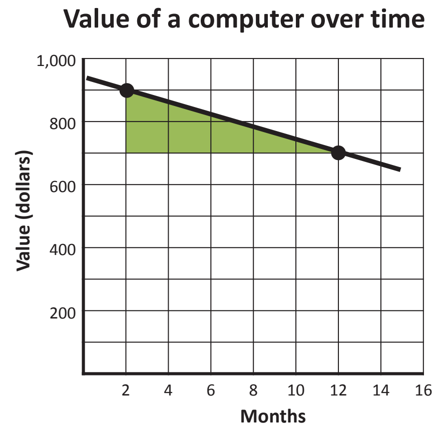
Students should recognize that any two points on the same line will generate the same rate of change (slope). In this example, the slope between points A and D is $\frac{9}{6}$, which simplifies to $\frac{3}{2}$.

(continued)

 * Used on STAAR

Instructional Implications (continued)

Determining the rate of change (slope) from a graph in a real-world context can be accomplished in the same manner; however, students must pay attention to the scale of the graph.



In the example shown at left about the value of a computer over time, counting along the gridlines would generate a rate of change of $-\frac{2}{5}$. However, since each *y*-interval on the graph represents $100 and each *x*-interval represents 2 months, the correct rate of change is $-\frac{2(100)}{5(2)} = -\frac{200}{10}$ (a decrease in value of $20 per month).

Instruction should also include determining the rate of change from algebraic representations (or linear equations). In mathematical contexts, when equations are written in slope-intercept form ($y = mx + b$), the determination is made by identifying the rate of change (or slope) as the coefficient of x (or m). When equations are written in standard form, students must transform the equation into slope-intercept form (by solving for y) to identify the rate of change. For example, the equation $4x - 2y = 10$ can be transformed to $y = 2x + 5$ in order to determine that the rate of change is 2.

In real-world contexts, students should also be able to interpret the units of the rate of change based on the variables used in the situation (y units per x units). For example, suppose the function $f(x) = 40x + 100$ gives the total service charge ($f(x)$ in dollars) for an electrician who is on a job for x hours. The rate of change (40) represents the increase in the service charge in dollars per hour.

Learning from Mistakes

Students may make the following mistakes:

- Switching values for *x* and *y* in the slope formula
- Making sign errors when computing rate of change (positive or negative)*
- Neglecting to note the scale when determining rate of change from a graphical representation*

Academic Vocabulary

rate of change*

Interesting Items

A.3(B) 2017 #26
A.3(B) 2017 #52

A.2(B)

A.2 Linear functions, equations, and inequalities. The student applies the mathematical process standards when using properties of linear functions to write and represent in multiple ways, with and without technology, linear equations, inequalities, and systems of equations. The student is expected to:

(B) write linear equations in two variables in various forms, including $y = mx + b$, $Ax + By = C$, and $y - y_1 = m(x - x_1)$, given one point and the slope and given two points

Stimulus

Word Problem	Verbal Description	Chart/Table	Graph
Equation/ Expression*	Manipulatives	Diagram/Image	Number Line
Base Ten Blocks	Measurement Tool	Formula	Geometric Figures

Academic Vocabulary

constant
linear equation
point
point-slope form (of a linear equation)
slope*
slope-intercept form (of a linear equation)
standard form* (of a linear equation)
y-intercept

Interesting Items

A.2(B) 2022 #25
A.2(B) 2016 #46

Role in Concept Development

Supports

A.2(C) write linear equations in two variables given a table of values, a graph, and a verbal description

Connection/ Relevance

Using the various forms of linear equations is essential for writing these equations from the information provided in a table, graph, or description.

When to Teach

With A.2(C)

Instructional Implications

Instruction should include writing the equation for a line in various forms. Given a table or graph, students identify two points and determine the slope using the formula $m = \frac{y_2 - y_1}{x_2 - x_1}$. With this value and one of the points, the equation for the line can be written in point-slope form using $y - y_1 = m(x - x_1)$. From this form, the linear equation can be rewritten in slope-intercept form ($y = mx + b$) or standard form ($Ax + By = C$). For example, consider the line that passes through the points (8, 1) and (-2, -4):

- To find the equation for the line, first find the slope. $m = \frac{-4-1}{-2-8} = \frac{-5}{-10} = \frac{1}{2}$
- Then, use $m = \frac{1}{2}$ and (8, 1) in the point-slope form of line. $y - 1 = \frac{1}{2}(x - 8)$
- Through distribution and adding 1 to both sides of this equation, it can be transformed to slope-intercept form. $y - 1 = \frac{1}{2}x - 4$; $y = \frac{1}{2}x - 3$
- By multiplying both sides by 2 and adding terms to both sides, the equation can be re-written in standard form. $2y = x - 6$; $2y + 6 = x$; $6 = x - 2y$ or $x - 2y = 6$

Learning from Mistakes

Students may make the following mistakes:
- Switching the *x* and *y* values in the slope formula
- Making sign errors when subtracting or distributing

A.2(E)

A.2 Linear functions, equations, and inequalities. The student applies the mathematical process standards when using properties of linear functions to write and represent in multiple ways, with and without technology, linear equations, inequalities, and systems of equations. The student is expected to:

(E) write the equation of a line that contains a given point and is parallel to a given line

Stimulus

Word Problem	Verbal Description	Chart/Table	Graph
Equation/ Expression	Manipulatives	Diagram/Image	Number Line
Base Ten Blocks	Measurement Tool	Formula	Geometric Figures

Academic Vocabulary

parallel*
slope
y-intercept*

Interesting Items

N/A

Role in Concept Development

Supports

- G.2(C) determine an equation of a line parallel or perpendicular to a given line that passes through a given point

Connection/ Relevance

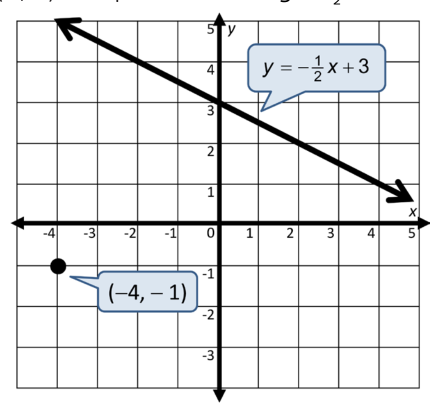
Students use their equation-writing skills in a specific context involving parallel lines. This skill will have continued relevance with the solving of systems of linear equations in which there is no solution.

When to Teach

Before/Prerequisite to G.2(C)

Instructional Implications

Instruction should include writing equations for parallel lines. In conjunction with A.2(B), students should be able to determine the slope of a line and write the equation of a line in point-slope and slope-intercept forms. In addition, students must also recognize that parallel lines have the same (equal) slopes.

Item	Solution
Write the equation for the line that passes through (-4, -1) and is parallel to the line $y=-\frac{1}{2}x+3$.  	• Identify the slope of the given line: $m=-\frac{1}{2}$ • Use this slope and the given point to write the equation of the parallel line in point-slope form: $y-(-1)=-\frac{1}{2}(x-(-4))$ • Simplify to rewrite the equation in slope-intercept form: $y+1=-\frac{1}{2}(x+4)$ $y+1=-\frac{1}{2}x-2$ $y=-\frac{1}{2}x-3$

Learning from Mistakes

Students may make the following mistakes:

- Failing to identify the correct slope
- Making sign errors or arithmetic mistakes when distributing and combining terms in changing from point-slope to slope-intercept form or vice versa*

A.2(F) **A.2 Linear functions, equations, and inequalities.** The student applies the mathematical process standards when using properties of linear functions to write and represent in multiple ways, with and without technology, linear equations, inequalities, and systems of equations. The student is expected to:

(F) write the equation of a line that contains a given point and is perpendicular to a given line

Stimulus

Word Problem	Verbal Description*	Chart/Table	Graph
Equation/ Expression*	Manipulatives	Diagram/Image	Number Line
Base Ten Blocks	Measurement Tool	Formula	Geometric Figures

Academic Vocabulary

linear equation
opposite reciprocal
perpendicular*
slope*

Interesting Items

A.2(F) 2019 #10

Role in Concept Development

Supports

- G.2(C) determine an equation of a line parallel or perpendicular to a given line that passes through a given point

Connection/ Relevance

Students use their equation-writing skills in a specific context involving perpendicular lines. This skill has continued relevance in the study of coordinate geometry in later courses.

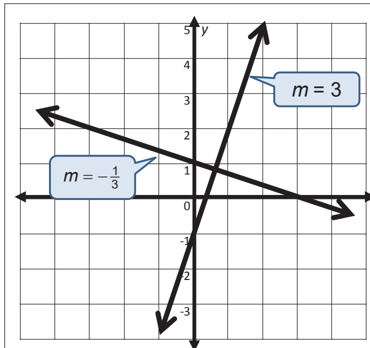
When to Teach

- Before/Prerequisite to G.2(C)

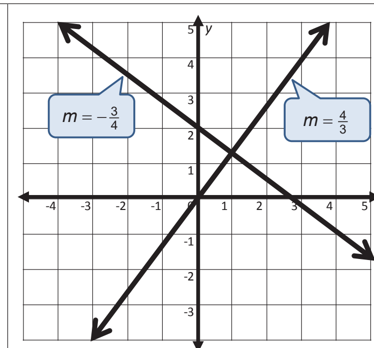
Instructional Implications

Instruction should include writing equations for perpendicular lines. In conjunction with A.2(B), students should be able to determine the slope of a line and write the equation of a line in point-slope and slope-intercept forms. Students must also recognize that perpendicular lines have slopes that are opposite reciprocals of each other (or slopes whose product equals -1). If a given line has a slope of $\frac{2}{3}$, then the line perpendicular to it will have a slope of $-\frac{3}{2}$.

Other examples of slopes of perpendicular lines are shown below.



Slopes of 3 and $-\frac{1}{3}$ create perpendicular lines.



Slopes of $-\frac{3}{4}$ and $\frac{4}{3}$ create perpendicular lines.

(continued)

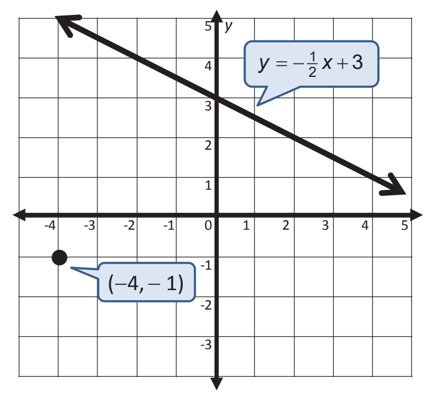
 * Used on STAAR

Role in Concept Development (continued)

Instructional Implications

Students are also expected to write an equation of a line that contains a given point and is perpendicular to a given line. For example:

<table>
<tr><th>Item</th><th>Solution</th></tr>
<tr><td>Write the equation for the line that passes through (-4, -1) and is perpendicular to the line $y = -\frac{1}{2}x + 3$.



</td><td>

- Identify the slope of the given line: $m = -\frac{1}{2}$
- Use the opposite reciprocal of this number to determine the slope of the perpendicular line: $m = 2$
- Use this slope and the given point to write the equation of the perpendicular line in point-slope form: $y-(-1)=2(x-(-4))$
- Simplify to rewrite the equation in slope intercept form: $y+1=2(x+4)$ $y+1=2x+8$ $y=2x+7$

</td></tr>
</table>

Learning from Mistakes

Students may make the following mistakes:

- Failing to identify the correct slope for the perpendicular line (e.g., neglecting to the take the opposite sign and/or the reciprocal of the original slope)
- Making sign errors or arithmetic mistakes when distributing and combining terms in changing from point-slope to slope-intercept form or vice versa*

A.2(G)

A.2 Linear functions, equations, and inequalities. The student applies the mathematical process standards when using properties of linear functions to write and represent in multiple ways, with and without technology, linear equations, inequalities, and systems of equations. The student is expected to:

(G) write an equation of a line that is parallel or perpendicular to the *x*- or *y*-axis and determine whether the slope of the line is zero or undefined

Stimulus

Word Problem	Verbal Description	Chart/Table	Graph*
Equation/ Expression*	Manipulatives	Diagram/Image	Number Line
Base Ten Blocks	Measurement Tool	Formula	Geometric Figures

Academic Vocabulary

horizontal
linear equation
parallel
perpendicular
slope*
undefined*
vertical
x-axis
y-axis

Interesting Items

A.2(G) 2018 #32
A.2(G) 2017 #36

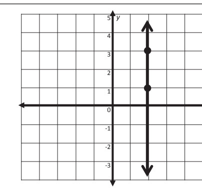
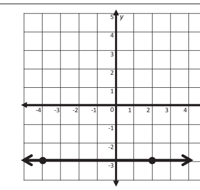
Role in Concept Development

Supports
A.2(C) write linear equations in two variables given a table of values, a graph, and a verbal description

Connection/ Relevance
Students write equations for lines and address the special cases when the lines are either vertical or horizontal.

When to Teach
With A.2(C)

Instructional Implications
Instruction should include writing equations for horizontal and vertical lines (parallel or perpendicular to the *x*- or *y*-axis) and determining their slopes. These equations take the form of $x = a$ or $y = b$ where a and b are real numbers. Students should be able to describe these types of equations by comparing their graphs, ordered pairs (or tables), equations, slopes, and relationships to the *x*-axis and *y*-axis.

Attribute	Vertical Lines	Horizontal Lines
Graph		
Ordered pairs	(2, 1), (2, 3), (The *x*-values are the same)	(–-4, -3), (2, -3), (The *y*-values are the same)
Equation	$x = 2$	$y = -3$
Slope	Undefined; $\frac{3-1}{2-2}=\frac{2}{0}$	Zero; $\frac{-3-(-3)}{2-(-4)}=\frac{0}{6}=0$
Type	Vertical line	Horizontal line
Is it a function?	No	Yes
Relationships to the *x*-axis	Perpendicular to the *x*-axis	Parallel to the *x*-axis
Relationship to the *y*-axis	Parallel to the *y*-axis	Perpendicular to the *y*-axis

Learning from Mistakes
Students may make the following mistakes:
- Switching the *x* and *y* values in the slope formula
- Mixing up or incorrectly identifying vertical and horizontal, undefined* and zero slope*, etc.

* Used on STAAR

A.3(A)

A.3 Linear functions, equations, and inequalities. The student applies the mathematical process standards when using graphs of linear functions, key features, and related transformations to represent in multiple ways and solve, with and without technology, equations, inequalities, and systems of equations. The student is expected to:

(A) determine the slope of a line given a table of values, a graph, two points on the line, and an equation written in various forms, including $y = mx + b$, $Ax + By = C$, and $y - y_1 = m(x - x_1)$

Stimulus

Word Problem	Verbal Description*	Chart/Table	Graph
Equation/ Expression*	Manipulatives	Diagram/Image	Number Line
Base Ten Blocks	Measurement Tool	Formula	Geometric Figures

Academic Vocabulary

linear equation
point-slope form (of a linear equation)
rate of change*
slope*
slope-intercept form (of a linear equation)
standard form (of a linear equation)

Interesting Items

N/A

Role in Concept Development

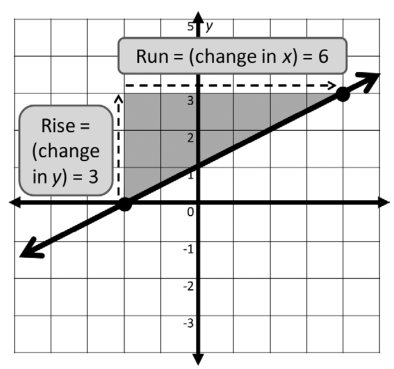
Supports: A.3(B) calculate the rate of change of a linear function represented tabularly, graphically, or algebraically in context of mathematical and real-world problems

Connection/ Relevance: This standard emphasizes the various ways students can identify the rate of change as slope using tables, graphs, and equations.

When to Teach: With A.3(B)

Instructional Implications: Instruction should include determining the slope of a line from various representations. When given a table of values for a linear function or two points on a line, students should determine the slope (m) from the formula $m = \frac{y_2 - y_1}{x_2 - x_1}$.

Representation	Slope
The table shows values for a linear function. x: 2, 6, 10 y: 9, 5, 1	$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{5 - 9}{6 - 2} = \frac{-4}{4} = -1$
A line contains the points (2, 3) and (-2, 9).	$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{9 - 3}{-2 - 2} = \frac{6}{-4} = -\frac{3}{2}$



From a graph, students determine the slope by counting horizontal and vertical units between two points on the line. The slope is defined as the ratio of the change in y to the change in x (or "rise over run"). In the example provided, since one point is 3 units above and 6 units to the right of the other, the slope is $\frac{3}{6} = \frac{1}{2}$.

(continued)

Role in Concept Development (continued)

Instructional Implications

When given an equation in the form $y = mx + b$ or $y - y_1 = m(x - x_1)$, students should be able to identify the slope as *m* (or the number in the equation that is multiplied by the *x* variable).

Representation	Slope
The equation for a line is $y = 3x + 7$.	The slope is 3.
The equation for a line is $y - 2 = -\frac{1}{3}(x + 5)$.	The slope is $-\frac{1}{3}$.

When given an equation in standard form ($Ax + By = C$), students must rewrite the equation in "*y* =" form to identify the slope. For equations of the form $Ax + By = C$, the slope can be identified as $m = -\frac{A}{B}$.

Representation	Slope
The equation for a line is $4x - 3y = 24$.	The equation can be rewritten as $y = \frac{4}{3}x - 8$, so the slope is $m = \frac{4}{3}$.

Learning from Mistakes

Students may make the following mistakes:
- Switching *x* and *y* values in the slope formula
- Dividing *y* and *x* values to determine slope (instead of dividing the differences)
- Identifying slope as the coefficient *A* in standard form (instead of *m* in slope-intercept form)*

A.4(C)

A.4 Linear functions, equations, and inequalities. The student applies the mathematical process standards to formulate statistical relationships and evaluate their reasonableness based on real-world data. The student is expected to:

(C) write, with and without technology, linear functions that provide a reasonable fit to data to estimate solutions and make predictions for real-world problems

Stimulus

Word Problem*	Verbal Description	Chart/Table*	Graph*
Equation/ Expression*	Manipulatives	Diagram/Image	Number Line
Base Ten Blocks	Measurement Tool	Formula	Geometric Figures

Academic Vocabulary

association/correlation
correlation coefficient
function*
linear function
linear regression
scatterplot*
strength (of correlation)

Interesting Items

A.4(C) 2019 #22
A.4(C) 2018 #12
A.4(C) 2016 #26

Role in Concept Development

Supports

- A.2(C) write linear equations in two variables given a table of values, a graph, and a verbal description
- 2A.8(C) predict and make decisions and critical judgments from a given set of data using linear, quadratic, and exponential models

Connection/ Relevance

Generating equations to match data helps connect algebraic skills to real-world situations. These skills are important for student success in later courses.

When to Teach

- With or After A.2(C)
- Before/Prerequisite to 2A.8(C)

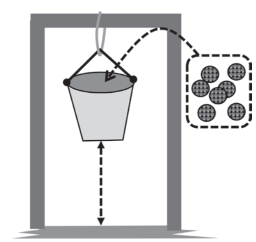
Instructional Implications

Instruction should include analyzing bivariate data collected from real-world situations. To model this data, students are expected to write a liner equation, with and without technology.

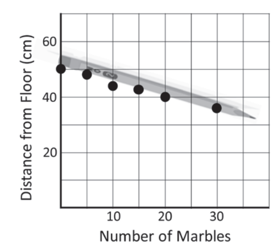
To estimate a line of best fit without technology, students should first construct a scatterplot. From this graphical representation, students can imagine placing a pencil or straw on the grid so that it is as close to as many points as possible. From this linear approximation, students can estimate ordered pairs to find the slope and write the equation for a line of fit [See A.2(B), A.2(C)].

For example, a paper cup is suspended by a rubber band (as shown below):

- As more marbles (x) are added to the cup, its distance from the floor (*y*, in cm) decreases.
- Students may draw a line of fit through (5, 48) and (30, 35). These points generate a slope of -0.52 and an equation of *y* = -0.52x + 50.6.



x	y
0	50
5	49
10	43
15	42
20	40
30	36



(continued)

Role in Concept Development (continued)

Instructional Implications

With technology, a student can enter the data into a calculator or other device to conduct a linear regression. For example, the data in the previous example would have the following regression output:

Linear Regression ($a \bullet x + b$)
a = -0.482857
b = 49.7714

Students would then generate the line of best fit as $y = -0.482857x + 49.7714$.

In either case, students can use the linear function to make predictions. For example, if asked to estimate how many marbles it would take until the cup was 20 cm from the ground, students would set $y = 20$ in either equation and solve for x.

Without Technology	With Technology
$y = -0.52x + 50.6$ $20 = -0.52x + 50.6$ $-30.6 = -0.52x$ $58.8462 = x$ (Or about 59 marbles)	$y = -0.482857x + 49.7714$ $20 = -0.482857x + 49.7714$ $-29.7714 = -0.482857x$ $61.6568 = x$ (Or about 62 marbles)

Learning from Mistakes

Students may make the following mistakes:
- Incorrectly identifying the slope and y-intercept from a regression equation*
- In making predictions, confusing or misinterpreting the x and y variables (whether to evaluate or solve)

A.12(C)

A.12 Number and algebraic methods. The student applies the mathematical process standards and algebraic methods to write, solve, analyze, and evaluate equations, relations, and functions. The student is expected to:

(C) identify terms of arithmetic and geometric **sequences when the sequences are given in function form using recursive processes**

Stimulus

Word Problem	Verbal Description	Chart/Table	Graph
Equation/ Expression	Manipulatives	Diagram/Image	Number Line
Base Ten Blocks	Measurement Tool	Formula	Geometric Figures

Academic Vocabulary

arithmetic sequence
common difference
common ratio
sequence
term

Interesting Items

N/A

Role in Concept Development

Supports

A.2(C) write linear equations in two variables given a table of values, a graph, and a verbal description

Connection/ Relevance

Arithmetic sequences create simple patterns that relate directly to linear functions. The relationship between the number of the term (*n*) and the term itself (a_n) mirrors the relationship between *x* and *y* (domain and range). The common difference of the sequence acts just like the slope of a linear function.

When to Teach

- With A.2(C), if teaching arithmetic sequences (only) in conjunction with linear functions
- After A.2(C), if teaching both arithmetic and geometric sequences together, after teaching linear and exponential functions separately

Instructional Implications

NOTE: The instructional considerations below provide guidance for using only arithmetic sequences (not geometric) when clustered with linear equations. For more information about geometric sequences, see A.12(C) in the Exponential Functions TEKS Cluster.

Instruction should include generating the terms of arithmetic sequences from a given function. In arithmetic sequences, terms increase/decrease by the same amount each time. This process can be repeated over and over (recursively).

The constant amount of increase is called the common difference. For example, in the arithmetic sequence 11, 9, 7, 5,..., the first term (a_1) is 11, the second term (a_2) is 9, etc., and the common difference is -2.

Students should be able to use a function to generate terms in a sequence. For example, consider the function $a(n) = 3n + 7$. If *n* represents the number of the term in a sequence, then evaluating at $n = 1, 2, 3...$ will generate the sequence $a_1 = 10$, $a_2 = 13$, $a_3 = 16$, etc. Recognizing an arithmetic sequence with a common difference of 3, students should be able to extend the sequence recursively by adding 3 each time (10, 13, 16, 19, 22, 25,...). Note also that the common difference of 3 corresponds to the slope of the related linear equation.

Learning from Mistakes

Students may make the following mistakes:

- Subtracting terms in the wrong order when computing the common difference*
- Confusing the number of the term (n) with the term itself (a_n)

* Used on STAAR

A.12(D) Supporting

A.12(D)

A.12 Number and algebraic methods. The student applies the mathematical process standards and algebraic methods to write, solve, analyze, and evaluate equations, relations, and functions. The student is expected to:

(D) write a formula for the n^{th} term of arithmetic and geometric **sequences, given the value of several of their terms**

Stimulus

Word Problem	Verbal Description	Chart/Table	Graph
Equation/ Expression*	Manipulatives	Diagram/Image	Number Line
Base Ten Blocks	Measurement Tool	Formula	Geometric Figures

Academic Vocabulary

arithmetic sequence
common difference
common ratio
sequence
term*

Interesting Items

A.12(D) 2018 #9

Role in Concept Development

Supports

A.2(C) write linear equations in two variables given a table of values, a graph, and a verbal description

Connection/ Relevance

Arithmetic sequences create simple patterns that relate directly to linear functions. The relationship between the number of the term (n) and the term itself (a_n) creates ordered pairs. For example, $a_1 = 4$ corresponds to (1, 4). Since the common difference of the sequence acts just like the slope of a linear function, the formula for finding the "n^{th}" term mirrors the point-slope formula for a line.

When to Teach

- With A.2(C), if teaching arithmetic sequences (only) in conjunction with linear functions
- After A.2(C), if teaching both arithmetic and geometric sequences together, after teaching linear and exponential functions separately

Instructional Implications

NOTE: The instructional considerations below provide guidance for using only arithmetic sequences (not geometric) when clustered with linear equations. For more information about geometric sequences, see A.12(D) in the Exponential Functions TEKS Cluster.

Instruction should include developing the formulas to find any term of an arithmetic sequence.

In arithmetic sequences, terms increase/decrease by the same amount each time. The constant amount of increase is called the common difference. For example, in the arithmetic sequence 11, 9, 7, 5,..., the common difference (d) can be found by subtracting any pair of consecutive terms. Here, the common difference is -2, since $a_2 – a_1 = 9 – 11 = -2$, or $a_3 – a_2 = 7 – 9 = -2$.

After defining the common difference, students should be asked how to find an unknown term in the sequence, such as the 50^{th} term. A sample response would be, "Start with the first term (11) and add the common difference (-2) 49 times." From this, students can develop the formula to find any term in an arithmetic sequence: $a(n) = a_1 + d(n – 1)$. Here, for example, the formula for the sequence 11, 9, 7, 5... would be $a(n) = 11 – 2(n - 1)$.

Learning from Mistakes

Students may make the following mistakes:

- Confusing the number of the term (n) with the term itself (a_n)
- Using the recursive formula instead of the explicit formula (e.g., given {2, 5, 8, 11, 14...} mistakenly writing the equation as $a_n = n + 3$)

TEKS Scaffold

TEKS	Student Expectation
2A.7(I)	write the domain and range of a function in interval notation, inequalities, and set notation (S)
A.2(A)	**A.2 Linear functions, equations, and inequalities.** The student applies the mathematical process standards when using properties of linear functions to write and represent in multiple ways, with and without technology, linear equations, inequalities, and systems of equations. The student is expected to: **(A) determine the domain and range of a linear function in mathematical problems; determine reasonable domain and range values for real-world situations, both continuous and discrete; and represent domain and range using inequalities**
8.5(G)	identify functions using sets of ordered pairs, tables, mappings, and graphs (R)

Stimulus

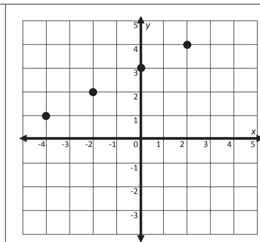
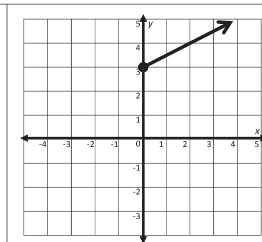
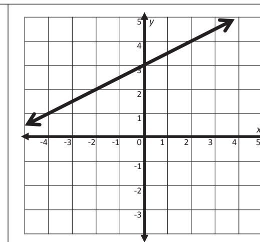
Word Problem*	Verbal Description*	Chart/Table	Graph*
Equation/ Expression*	Manipulatives	Diagram/Image	Number Line
Base Ten Blocks	Measurement Tool	Formula	Geometric Figures

Content Builder (see Appendix for Tree Diagram)

- Determine the domain of a linear function in mathematical problems
- Determine the range of a linear function in mathematical problems
- Determine reasonable continuous domain values for real-world situations
- Determine reasonable discrete domain values for real-world situations
- Determine reasonable continuous range values for real-world situations
- Determine reasonable discrete range values for real-world situations
- Represent domain and range using inequalities

Instructional Implications

Students are expected to determine domain and range values for linear functions in a variety of contexts using various representations. Instruction should begin by defining domain as the set of *x*-values used by a function and range as the set of *y*-values. Students determine domain and range from graphs. Examples should include graphs of linear functions that are both discrete and continuous, since the type of function will determine whether the domain and range can be described using lists in set notation (for discrete functions) or using inequalities (for continuous functions).

Graph			
Function type	Discrete	Continuous	Continuous
Domain	{-4, -2, 0, 2}	$x \geq 0$	All real numbers
Range	{1, 2, 3, 4}	$y \geq 3$	All real numbers

(continued)

Learning from Mistakes

Students may make the following mistakes:

- Confusing *x* and *y* values (domain and range)*
- Confusing which inequality symbol to use (< or >, > or ≥, etc.) to represent domain and range
- Confusing which inequality symbol to use (< or >, > or ≥, etc.) to represent the inclusive (closed circle) and exclusive data (open circle)*
- Having trouble recognizing whether a real-world situation should be represented with discrete or continuous variables*
- Confusing domain and range on the graph (e.g., seeing domain as the "height" of the graph instead of the "width")

Academic Vocabulary

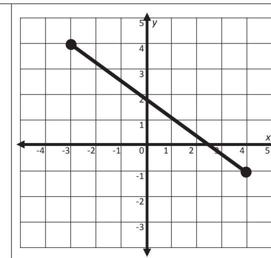
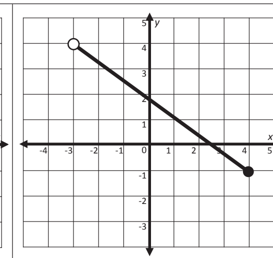
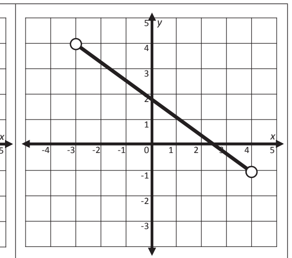
continuous
discrete
domain*
function*
inequality*
linear function*
range*

Interesting Items

A.2(A) 2022 #18
A.2(A) 2021 #33
A.2(A) 2018 #13
A.2(A) 2016 #30
A.2(A) 2016 #44

Instructional Implications (continued)

Examples should also include continuous linear functions with closed and open circles as endpoints. Closed circles include the endpoints and use the symbols ≤ or ≥. Open circles do not include the endpoints and use the symbols < or >.

Graph			
Domain	-3 ≤ x ≤ 4 The domain includes both -3 and 4.	-3 < x ≤ 4 The domain includes 4, but not -3.	-3 < x < 4 The domain includes values of *x* strictly between -3 and 4 (not inclusive)

In addition to identifying the domain and range of a graph, students are expected to interpret limitations on independent and dependent variables in real-world problems. For example, certain situations may require the use of only positive numbers or only whole numbers, which restrict the values in the domain and/or range.

Sample Situation	The French Club will wash cars for $5 each to raise money. Their profit is determined by taking this revenue and subtracting $15 for the supplies they purchased.	A plumber's total charge is $60 for a home visit, plus $40 per hour.
Function	*f(x)* = 5*x* – 15	*f(x)* = 40*x* + 60
Domain	{0, 1, 2, 3, 4, 5 ...} The Club members can only wash a whole number of cars.	x ≥ 0 Hours must be 0 or more, but can include fractions of an hour.
Range	{-15, -10, -5, 0, 5, 10 ...} The Club's profit can be as low as -$15, but will increase at multiples of $5.	*f(x)* ≥ 60 The least charge is $60, and can increase from there.

TEKS Scaffold

TEKS	Student Expectation
2A.2(A)	graph the functions $f(x) = \sqrt{x}$, $f(x) = 1/x$, $f(x) = x^3$, $f(x) = \sqrt[3]{x}$, $f(x) = b^x$, $f(x) = \lvert x \rvert$, and $f(x) = \log_b (x)$ where *b* is 2, 10, and *e*, and, when applicable, analyze the key attributes such as domain, range, intercepts, symmetries, asymptotic behavior, and maximum and minimum given an interval (R)
A.3(C)	**A.3 Linear functions, equations, and inequalities.** The student applies the mathematical process standards when using graphs of linear functions, key features, and related transformations to represent in multiple ways and solve, with and without technology, equations, inequalities, and systems of equations. The student is expected to: **(C) graph linear functions on the coordinate plane and identify key features, including *x*-intercept, *y*-intercept, zeros, and slope, in mathematical and real-world problems**
8.4(C)	use data from a table or graph to determine the rate of change or slope and *y*-intercept in mathematical and real-world problems (R)
8.4(B)	graph proportional relationships, interpreting the unit rate as the slope of the line that models the relationship (R)
7.7(A)	represent linear relationships using verbal descriptions, tables, graphs, and equations that simplify to the form $y = mx + b$ (R)

Stimulus

Word Problem*	Verbal Description	Chart/Table	Graph*
Equation/ Expression*	Manipulatives	Diagram/Image	Number Line
Base Ten Blocks	Measurement Tool	Formula	Geometric Figures

Content Builder (see Appendix for Tree Diagram)

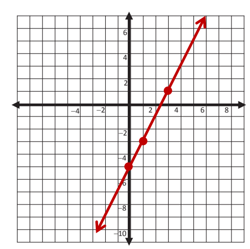
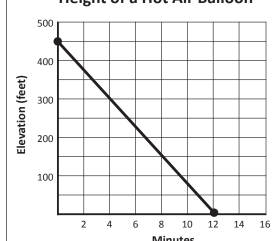
- Graph linear functions on the coordinate plane
- Identify the *x*-intercept of a linear function in mathematical problems
- Identify the *x*-intercept of a linear function in real-world problems
- Identify the *y*-intercept of a linear function in mathematical problems
- Identify the *y*-intercept of a linear function in real-world problems
- Identify the zeros of a linear function in mathematical problems
- Identify the zeros of a linear function in real-world problems
- Identify the slope of a linear function in mathematical problems
- Identify the slope of a linear function in real-world problems

Instructional Implications

Students should graph linear functions on the coordinate plane. Strategies for graphing may depend on the function's form. For equations written in $y = mx + b$ form, students can graph using technology, making a table and plotting points, or using the *y*-intercept and slope. For equations written in standard form, students can graph using the intercept method.

From a linear function and its graph, students must be able to identify the following key features:
- *x*-intercept – the point where the line crosses the *x*-axis
- *y*-intercept – the point where the line crosses the *y*-axis
- zero – the value of *x* that makes $y = 0$
- slope – the line's rate of change, measured as the change in *y* over the change in *x* [see A.3(B)]

In addition to identifying these features in a mathematical context, students must also be able to interpret their meaning in real-world problems. Examples of each type are included below.

<table>
<tr><th></th><th>Mathematical Context</th><th colspan="2">Real-world Problem</th></tr>
<tr><td>Function</td><td>$y = 2x – 5$</td><td>$f(x) = 450 – 37.5x$</td><td rowspan="2">A hot air balloon is spotted at a height of 450 ft. As it makes its descent back to the ground, its height is a linear function of time in minutes.</td></tr>
<tr><td>Graph</td><td></td><td>Height of a Hot Air Balloon
</td></tr>
<tr><td>Slope</td><td>2</td><td colspan="2">-37.5, which means that the balloon descends at a rate of 37.5 ft/min.</td></tr>
<tr><td>x-intercept</td><td>(2.5, 0)</td><td>(12, 0)</td><td rowspan="2">After 12 seconds, the balloon will be on the ground (or, its height will be 0 ft).</td></tr>
<tr><td>Zero</td><td>$x = 2.5$</td><td>$x = 12$ seconds</td></tr>
<tr><td>y-intercept</td><td>(0, -5)</td><td colspan="2">(0, 450) At the moment it was spotted (after 0 sec, or when $x = 0$), the balloon was 450 ft high.</td></tr>
</table>

 * Used on STAAR

Learning from Mistakes

Students may make the following mistakes:

- Switching values for the change in *x* and the change in *y* when identifying slope
- Making sign errors (positive or negative) when identifying slope*
- Confusing *x*-intercepts and *y*-intercepts
- Identifying slope as the coefficient *A* in standard form (instead of *m* in slope-intercept form)*

Academic Vocabulary

linear function*
slope
x-intercept*
y-intercept*
zeros*

Interesting Items

A.3(C) 2019 #26
A.3(C) 2017 #12

A.3(E)

A.3 Linear functions, equations, and inequalities. The student applies the mathematical process standards when using graphs of linear functions, key features, and related transformations to represent in multiple ways and solve, with and without technology, equations, inequalities, and systems of equations. The student is expected to:

(E) determine the effects on the graph of the parent function *f(x)* = *x* when *f(x)* is replaced by *af(x)*, *f(x)* + *d*, *f(x – c)*, *f(bx)* for specific values of *a*, *b*, *c*, and *d*

Stimulus

Word Problem	Verbal Description*	Chart/Table	Graph*
Equation/ Expression*	Manipulatives	Diagram/Image	Number Line
Base Ten Blocks	Measurement Tool	Formula	Geometric Figures

Academic Vocabulary

linear function*
parent function
slope*
transformation
translation (up, down, right, left)
y-intercept*

Interesting Items

A.3(E) 2021 #20
A.3(E) 2019 #52
A.3(E) 2016 #11

Role in Concept Development

Supports

- A.3(C) graph linear functions on the coordinate plane and identify key features, including *x*-intercept, *y*-intercept, zeros, and slope, in mathematical and real-world problems
- A.7(C) determine the effects on the graph of the parent function $f(x) = x^2$ when *f(x)* is replaced by *af(x)*, *f(x)* + *d*, *f(x – c)*, *f(bx)* for specific values of *a*, *b*, *c*, and *d*

Connection/ Relevance

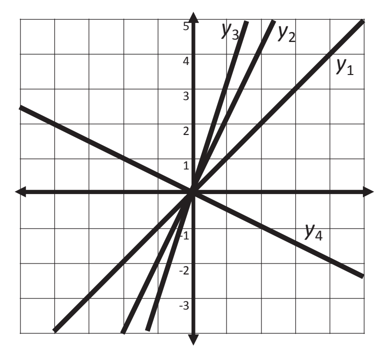
This standard provides a visual (graphical) way to describe and define slope and *y*-intercept of linear functions through transformations. Specifically, changing *a* in *f(x)* = *ax* affects the slope and changing *d* in *f(x)* = *x* + *d* changes the intercepts. This study of transformation will then be applied to quadratic equations.

When to Teach

- With A.3(C)
- Before/Prerequisite to A.7(C)

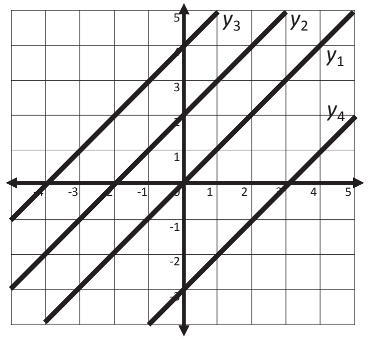
Instructional Implications

Instruction should include analysis of the graph of the linear parent function (*f(x)* = x) when the equation is changed by adding, subtracting, or multiplying constants by the *x*-variable. Students may explore these transformations using technology, where they graph several related functions and are asked to explain the connections between the functions and the graphs.

Functions	Graphs	Explanation
$y^1 = x$ $y^2 = 2x$ $y^3 = 3x$ $y^4 = -0.5x$		Multiplying *x* by different values changes the steepness of the line. Negative coefficients make the line go down (from left to right).

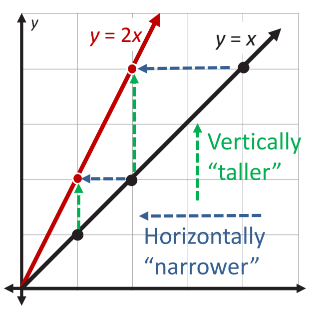
(continued)

Role in Concept Development (continued)

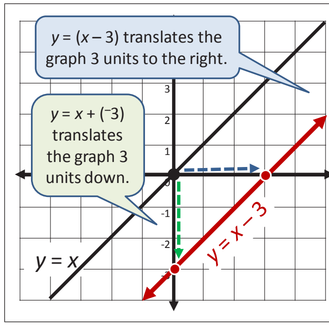
Functions	Graphs	Explanation
$y^1 = x$ $y^2 = x + 2$ $y^3 = x + 4$ $y^4 = x - 3$		Adding different values to *x* translates the graph up. Subtracting values translates the graph down.

From these explorations, students should relate these transformations to changes in the linear parent function's slope and *y*-intercept.

Note that, for the linear parent function, the coefficients *a* and *b* (in *f(x)* = *a* • *x* and *f(x)* = *(bx)*) generate identical transformations by changing the slope.

	These transformations can be described as a vertical change (For *y* = *ax, a* = 2 makes the graph of the parent function "taller") or as a horizontal change (For *y* = *(bx), b* = 2 makes the graph of the parent function "narrower").

Likewise, the constants *c* and *d* (in *f(x)* = *x* – *c* and *f(x)* = *x* + *d*) both affect the linear parent function's *y*-intercept.

	c translates the parent function horizontally (to the right when *c* > 0, or left when *c* < 0), while d translates the parent function vertically (up when *d* > 0, or down when *d* < 0).

These subtle differences (between *a* and *b*, and between *c* and *d*) are more obvious when analyzing transformations of different parent functions, such as $f(x) = x^2$ [see A.7(C)].

Learning from Mistakes

Students may make the following mistakes:

- Describing the transformation incorrectly, possibly by mis-identifying the original and new functions*

A.4(A)

A.4 Linear functions, equations, and inequalities. The student applies the mathematical process standards to formulate statistical relationships and evaluate their reasonableness based on real-world data. The student is expected to:

(A) calculate, using technology, the correlation coefficient between two quantitative variables and interpret this quantity as a measure of the strength of the linear association

Stimulus

Word Problem	Verbal Description*	Chart/Table*	Graph
Equation/ Expression	Manipulatives	Diagram/Image	Number Line
Base Ten Blocks	Measurement Tool	Formula	Geometric Figures

Academic Vocabulary

association/correlation
correlation coefficient
linear function
linear regression
strength (of correlation)

Interesting Items

A.4(A) 2017 #19

Role in Concept Development

Supports

- A.2(C) write linear equations in two variables given a table of values, a graph, and a verbal description
- 2A.8(C) predict and make decisions and critical judgments from a given set of data using linear, quadratic, and exponential models

Connection/ Relevance

Using technology to generate regression equations and correlation coefficients helps to connect writing equations in purely mathematical context to situations involving real-world data. These skills are important for student success in later courses.

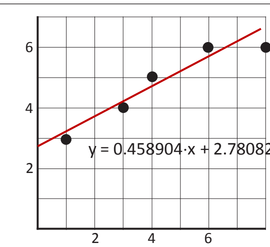
When to Teach

- After A.2(C)
- Before/Prerequisite to 2A.8(C)

Instructional Implications

Instruction should include using technology to calculate a regression equation to model a set of data. This process also generates a correlation coefficient, which students should identify as a measure of the strength of the linear association.

The first step involves inputting ordered pairs of data into a calculator or other device, then running a linear regression on these data. It may be helpful for students to view a scatterplot of the data along with the regression equation.

<table>
<tr><th>Data</th><th>Regression Output</th><th>Graph</th></tr>
<tr><td><table><tr><th>x</th><th>y</th></tr><tr><td>1</td><td>3</td></tr><tr><td>3</td><td>4</td></tr><tr><td>4</td><td>5</td></tr><tr><td>6</td><td>6</td></tr><tr><td>8</td><td>6</td></tr></table></td><td>Linear Regression ($a \bullet x + b$)
a = 0.458904
b = 2.78082
r^2 = 0.904311
r = 0.950953</td><td>
</td></tr>
</table>

(continued)

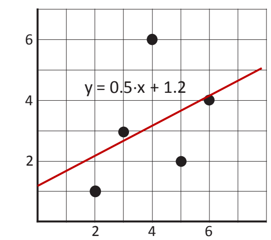
 * Used on STAAR

Role in Concept Development (continued)

Instructional Implications

Data	Regression Output	Graph
(table below)	Linear Regression (*a* • *x*+*b*) a = 0.5 b = 1.2 r^2 = 0.168918 r = 0.410997	(graph below)

x	*y*
2	1
3	3
4	6
5	2
6	4



In each case, the correlation coefficient is the value labeled as *r*. Values of *r* that are close to 1 or -1 indicate a strong linear association, whereas values that are closer to 0 indicate a weaker association. In the examples provided, the first data set (*r* = 0.950953) has a much stronger linear association than the second data set (*r* = 0.410997).

Learning from Mistakes

Students may make the following mistakes:
- Switching the values of the slope and *y*-intercept from the regression equation
- Failing to identify strong correlations (close to 1 or -1, instead of close to zero)

A.4(B)

A.4 Linear functions, equations, and inequalities. The student applies the mathematical process standards to formulate statistical relationships and evaluate their reasonableness based on real-world data. The student is expected to:

(B) compare and contrast association and causation in real-world problems

Stimulus

Word Problem	Verbal Description*	Chart/Table	Graph
Equation/ Expression	Manipulatives	Diagram/Image	Number Line
Base Ten Blocks	Measurement Tool	Formula	Geometric Figures

Academic Vocabulary

association/correlation
causation*
linear function

Interesting Items

A.4(B) 2017 #9

Role in Concept Development

Supports

- A.3(B) calculate the rate of change of a linear function represented tabularly, graphically, or algebraically in context of mathematical and real-world problems
- A.3(C) graph linear functions on the coordinate plane and identify key features, including *x*-intercept, *y*-intercept, zeros, and slope, in mathematical and real-world problems

Connection/ Relevance

Comparisons between association and causation help connect real-world situations to algebraic relationships (like rate of change and intercepts) in linear functions.

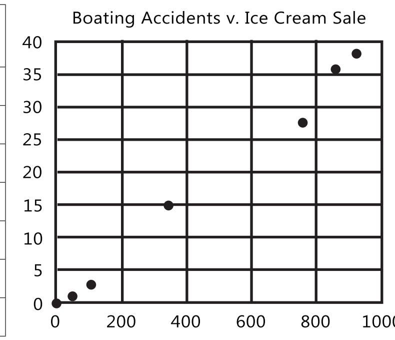
When to Teach

After A.3(B) and A.3(C)

Instructional Implications

Students should recognize that two quantities can exhibit a correlation (association) without being causally related (causation)

Month	Ice Cream Sales ($)	Boating Accidents
Feb	0	0
March	54	1
Apr	110	3
May	350	15
Jun	754	28
Jul	855	36
Aug	922	38



While there is obviously an association between ice cream sales and the number of boating accidents (as one increases, so does the other), it is incorrect to assume causation (that an increase in ice cream sales causes boating accidents or that an increase in boating accidents causes people to buy more ice cream). The increase in both quantities may actually be caused by other variables that were not mentioned (such as temperature, the number of tourists at the resort, etc.).

Learning from Mistakes

Students may make the following mistakes:

- Incorrectly identifying independent and dependent variables
- Misinterpreting the relationship between the variables*
- Confusing association with causation*

A.12(A)

A.12 Number and algebraic methods. The student applies the mathematical process standards and algebraic methods to write, solve, analyze, and evaluate equations, relations, and functions. The student is expected to:

(A) decide whether relations represented verbally, tabularly, graphically, and symbolically define a function

Stimulus

Word Problem	Verbal Description	Chart/Table*	Graph
Equation/ Expression	Manipulatives	Diagram/Image	Number Line
Base Ten Blocks	Measurement Tool	Formula	Geometric Figures

Academic Vocabulary

domain
range

Interesting Items

A.12(A) 2016 #36

Role in Concept Development

Supports

A.2(A) determine the domain and range of a linear function in mathematical problems; determine reasonable domain and range values for real-world situations, both continuous and discrete; and represent domain and range using inequalities

Connection/ Relevance

Much of Algebra I is based on the identification of functions and functional relationships. Students must be able to identify the relationship between the domain and range of linear functions in order to appropriately write, solve, analyze and evaluate functions.

When to Teach

Before/Prerequisite to A.2(A) and A.6(A)

Instructional Implications

Instruction should include defining a function as a relation where each element in the domain is paired with exactly one element in the range and applying this definition in multiple representations.

Students should be able to identify functions from a table of *x*- and *y*-values. If a relation is a function, then no *x*-value will be repeated in the table.

This is NOT a function:

x	y
5	1
6	2
7	3
6	4

(An *x*-value repeats; for $x = 6$, there are two possible *y*-values.)

This IS a function:

x	y
1	5
2	6
3	7
4	6

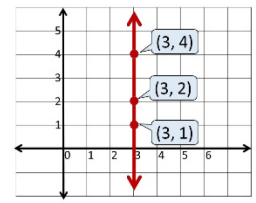
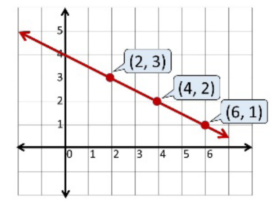
(No *x*-value repeats; there is only one *y*-value for each *x*.)

(continued)

Role in Concept Development (continued)

Instructional Implications

Students should also be able to recognize a function from the graph. On the coordinate plane, functions will pass the "vertical line test," which means that any vertical line will intersect the graph of a function at no more than one point.

In the cluster for linear functions, students should recognize that vertical lines are NOT functions because of the repeated x-values. Other lines with defined slopes ARE functions, including horizontal lines.	This is <u>NOT</u> a function:  A vertical line is NOT a function (*x*-values repeat)	This is <u>IS</u> a function:  A line with a slope IS a function (x-values will not repeat)

Symbolically, relations are functions if they can be written as a single equation in "y =" form. In the cluster for linear functions, an equation such as $4x + 2y = 10$ is a function because it can be rewritten as $y = -2x + 5$. Since vertical lines are not functions, they cannot be written in "y=" form. Specifically, equations for vertical lines are of the form $x = a$, such as $x = 3$.

Students should be able to identify whether a relation is a function from a verbal description by applying the definition to the *x*- and *y*-variables represented in the situation. For example:

Description	In a high school, each student's grade level (*x*) is paired with the student's height (*y*) in inches.	In a high school, each student's ID number (*x*) is paired with the student's grade level (*y*).
Is it a function?	NO	YES
Reason	Many students will be in the same grade but have different heights. So, the same *x*-value will be paired with many different *y*-values.	Each student has a unique ID number that will not be repeated, and a student will not be classified in more than one grade level.

Learning from Mistakes

Students may make the following mistakes:
- Switching *x* and *y* values (or switching domain and range)
- Confusing the rule about repeated *y*-values (which can be a function) with repeated *x*-values (which is not a function)*

* Used on STAAR

A.12(B) Supporting

A.12(B)

A.12 Number and algebraic methods. The student applies the mathematical process standards and algebraic methods to write, solve, analyze, and evaluate equations, relations, and functions. The student is expected to:

(B) evaluate functions, expressed in function notation, given one or more elements in their domains

Stimulus

Word Problem	Verbal Description	Chart/Table	Graph
Equation/ Expression*	Manipulatives	Diagram/Image	Number Line
Base Ten Blocks	Measurement Tool	Formula	Geometric Figures

Academic Vocabulary

domain
function*
function notation
range

Interesting Items

A.12(B) 2016 #2

Role in Concept Development

Supports

A.2(A) determine the domain and range of a linear function in mathematical problems; determine reasonable domain and range values for real-world situations, both continuous and discrete; and represent domain and range using inequalities

Connection/ Relevance

Understanding function notation is important for high school mathematics both symbolically (for writing equations and functions) and conceptually (to understand the input/output relationship between the domain and range of functions).

When to Teach

With A.2(A)

Instructional Implications

Instruction should include the use of function notation to describe the process of evaluating a function with values from its domain. For example, consider the function $f(x) = 2x - 3$. When given an expression such as $f(4)$, students should know that this means to evaluate the function $f(x)$ when $x = 4$ (or substitute $x = 4$ into the expression). Here, $f(4) = 2(4) - 3 = 8 - 3 = 5$. Students could summarize by writing $f(4) = 5$ (read as "*f* of 4 equals 5"). Students should recognize that $f(4) = 5$ indicates that a domain value of 4 results in a range value of 5.

Students may also recognize that function notation provides a more efficient way of writing problems with x's and y's. For example, see the comparison chart below.

Notation	Function notation	"$y =$" notation
Example	$f(x) = 2x - 3$	$y = 2x - 3$
Item	Find $f(4)$.	Find y when $x = 4$.
Answer	$f(4) = 205$	When $x = 4$, $y = 5$.

Learning from Mistakes

Students may make the following mistakes:

- Making sign errors or arithmetic mistakes in evaluating expressions
- After evaluating a function, thinking that this requires "solving for *f*" (e.g., given the expression $f(4) = 20$, thinking they need to divide both sides by 4 to "solve for *f*")