Students require frequent opportunities to engage in the problem-solving process to build mathematical proficiency.

The process standards are applied in every unit of study and are consistent across grade levels. The more independent students are with the process standards, the more confident they will be as learners.

We developed some strategies to help students understand, practice, and integrate these critical standards as they think about and apply what they learn.

The *Instructional Strategies Playlist for the Process Standards* is organized into two sections: tools to know (getting started) and ways to show (application). Choose a strategy based on the process standard and purpose you want students to practice. (Clicking on the strategy will take you directly to that page.)

These math-specific instructional strategies can be used with any unit of instruction or grade level!

process	purpose	strategy
	analyze information, formulate a plan/ strategy, estimate	<u>Comprehension</u> <u>Tower</u>
	analyze information	<u>Graffiti Art</u>
apply math in everyday situations and	analyze information, formulate a plan/ strategy	Justified List
use problem-solving models	analyze information, formulate a plan/ strategy, estimate	Positive Pings
	analyze information, formulate a plan/ strategy	Rock and Roll Item Review
	analyze information, formulate a plan/ strategy, estimate	Toss a Question

tools to know

ways to show

process	purpose	strategy
	communicate understanding, reasonableness, and conjectures	<u>Comprehension</u> <u>Tower</u>
	communicate understanding and analyze connections	Ditto Dance Off
create	communicate understanding using multiple representations and analyze connections	<u>Graffiti Art</u>
and analyze information	communicate understanding using multiple representations and analyze connections	Justified List
	communicate understanding using multiple representations and make connections	Positive Pings
	communicate understanding using multiple representations, reasonableness, conjectures, and connections	<u>Rock and Roll</u> Item Review

Comprehension Tower

Purpose

analyze information, formulate a plan/strategy, estimate

Instructions

- 1. Organize students into small groups.
- 2. Provide each group the following:
 - Set of blocks marked with four different colors
 - Comprehension Tower template
 - Set of math problems
 - Comprehension Tower recording sheet (optional)
- 3. Students participate in rock, paper, scissors to determine who in the group makes the first move. Moving blocks consists of taking one block from any level (except the one below the incomplete top level) of the tower. Only one hand should be used at a time when taking blocks from the tower. Blocks may be bumped to find a loose block that will not disturb the rest of the tower. Any block that is moved out of place must be returned to its original location before attempting to remove another block.
- 4. Students take turns removing a block, selecting a math problem from the pile of questions, and answering the associated color-coded question from the Comprehension Tower template. Teacher may elect to have students record their responses on the Comprehension Tower recording sheet.
- 5. Students return the question cards to the bottom of the stack but keep the blocks they removed from the tower.
- 6. The activity ends when the tower falls. Points may be assigned to each color-coded block (e.g., blue = 95 / 2.3 / -8; green = 150 / 1.5 / -5; red = 75 / ³/₄ / 7; purple = 85 / 2¹/₂ / -4). Students with the most points win the activity.

Materials

- Set of blocks marked with four different colors on the end
- Comprehension Tower Tools to Know template (color correlated to blocks)
- Set of math problems
- Comprehension Tower recording sheet (optional)

tools to know

Comprehension Tower Color-Correlated Blocks and Tools to Know Template



Identify three important details about the problem

Explain what information is not important to the problem

Represent the problem with an image (picture, strip diagram, array, table, equation, etc.)

Estimate a solution that would be too high and too low

Comprehension Tower – Sample Math Problems

#1 A water dispenser contains 512 fluid ounces of water. What is the total number of 8-fluid-ounce cups of water that can be filled from the dispenser?	 #2 Sabra read a total of 185 pages in three days. One the first day, she read 85 pages On the second and third days, she read the same number of pages How many pages did she read on the third day?
#3 Mark had 45 football cards. Josh had twice as many football cards as Mark. Josh then bought 5 more football cards. How many football cards does Josh have now?	#4 A factory makes 400 refrigerators every day. The factory makes 125 more stoves per day than refrigerators. What is the total number of refrigerators and stoves the factory makes per day?
#5 Diane worked 18 hours each week during the summer. She worked a total of 8 weeks and earned \$9 an hour. How much money did Diane earn during the summer?	#6 Terrell spent \$306 on a television and 3 video games. He spent \$243 on the television. Each video game was the same price. How much did Terrell spend on each video game?

math

instructional strategies playlist – process standards

Comprehension Tower – Recording Sheet

	estimate solutions						
	estima						
	represent the problem						
	explain what's not important						
•	3 important details						
	Question Number	1	2	n	4	ъ	9

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Graffiti Art

Purpose

analyze information

Instructions

- 1. Organize students into groups.
- 2. Give each group a different colored marker.
- 3. Groups rotate through stations to observe a given assessment stimuli/image (e.g., graph, table, equation, geometric shape, clock, thermometer, etc.) on poster/chart paper.
- 4. Students record on the poster/chart paper any information they know about the given stimuli.
- 5. When prompted, groups rotate to the next poster, read the previously written statements about the given stimuli, and add more details that were not thought of by the previous group(s).
- 6. Students continue to rotate through the remaining stations until they return to their original station.
- 7. Students analyze the various statements for accuracy and summarize their findings to share with the class.

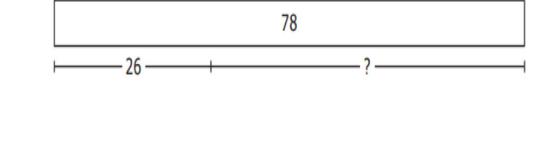
Materials

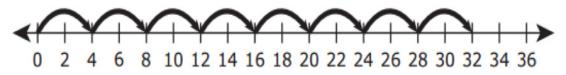
- Stimuli/Images from assessment items
- Poster/chart paper
- Markers

Think It Up

Have groups rotate through each station again, recording different questions that could have been asked through the lens of the given stimuli.

Graffiti Art – Elementary Examples (put each image on its own poster/chart paper)





Canoes and Paddleboats

Number of Canoes	Number of Paddleboats
72	90
37	55
61	79
85	103
68	86

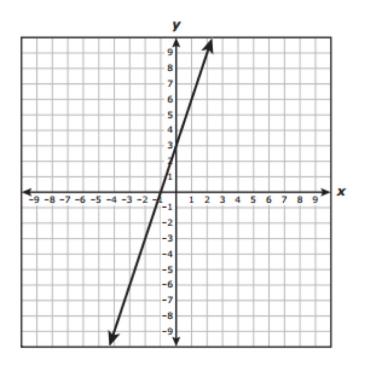
The members of a gym use 98 towels every day. How many towels are used in 7 days?

Graffiti Art – Secondary Examples (put each image on its own poster/chart paper)

Caroline's Cell Phone Plan

Month	1	2	3	4
Total Amount Paid	\$32	\$64	\$96	\$128

Frank and his family drove 6 hours every day during a road trip.



y = 0.5x + 14

Justified List

Purpose

analyze information, formulate a plan/strategy

Instructions

- 1. Give each student a Justified List recording sheet. The recording sheet includes a question and associated true/false statements to support students in practicing how to get started (tools to know) with the math problem.
- 2. Students place a check mark next to all the items that are true statements associated with the question and justify individual responses.
- 3. Students place an "X" next to all the items that are false statements associated with the question and justify individual responses.
- 4. Organize students into small groups.
- 5. Students engage in a Learning Loop (see Movement and Discourse Playlist) to share their thoughts.
- 6. Teacher sees and hears students' thinking and clarifies/verifies as appropriate.

Materials

• Justified List recording sheet

Justified List Recording Sheet – Elementary Example

A factory makes 40 refrigerators every day. The factory makes 12 more stoves per day than refrigerators. How many stoves and refrigerators were made in three days?

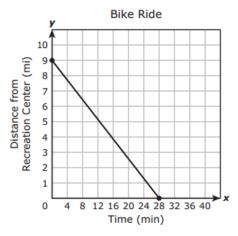
Place a check mark next to the statements that are true and an "X" next to the statements that are false. Justify your reasoning.

Statements	Justification
 The factory makes more refrigerators than stoves. 	
2. The factory makes 28 refrigerators per day.	
3. The factory makes 52 stoves per day.	
 4. The strip diagram below represents the total number of refrigerators and stoves the factory makes in one day. Stoves Refrigerators 40 40 12 	e
 5. This will be a multi-step problem.	

Justified List Recording Sheet – Secondary Example

A student rode a bike from school to a recreation center. The graph shows the student's distance in miles from the recreation center after riding the bike for x minutes.

Place a check mark next to the statements that are true and an "X" next to the statements that are false. Justify your reasoning.



	Statements	Justification
1.	This is a linear function.	
2.	The data on the <i>x</i> -axis represents the amount of time traveled in minutes.	
3.	The data on the <i>y</i> -axis represents the number of miles traveled.	
4.	The dependent variable is time (minutes).	
5.	The domain is all of the set of input/x-values on the graph.	
6.	The range is the total number of minutes it took the student to travel.	
7.	The <i>y</i> -intercept of (0,9) represents that the student was 9 miles away from the recreation center when he started the bike ride.	
8.	The <i>x</i> -intercept of (0,28) represents that the student took 28 minutes to arrive at the recreation center.	

Positive Pings

Purpose

analyze information, formulate a plan/strategy, estimate

Instructions

- 1. Organize students into small groups.
- 2. Provide each group with a large plastic cup and a ping pong ball.
- 3. Taking turns, each student bounces a ping pong ball off the table in an attempt to make it in the cup. Give each student six attempts. Repeat this step for each group member.
- 4. Students answer questions about a math problem/assessment item based on their number of positive pings. For example:
 - 0-1 Give three important details about the problem.
 - 2-3 Identify a strategy that would help you get started with the problem.
 - 4-5 Explain in your own words what the problem is wanting you to solve.
 - 6 Estimate a reasonable solution.
- 5. As each group member shares their responses, other group members record the responses on their Positive Ping recording sheet or notebook paper/journal.
- 6. Model the problem-solving process to show students the solution to the math problem.

Materials

- Large plastic cups and ping pong balls
- Math problem/assessment item
- Positive Ping recording sheet (optional)

tools to know

Positive Pings – Recording Sheet

Give three important details about the problem.	Identify a strategy that would help you get started with the problem.
1)	
2)	
3)	
Explain in your own words what the	Estimate a reasonable solution.
problem is wanting you to solve.	

Rock and Roll Item Review

Purpose

analyze information, formulate a plan/strategy

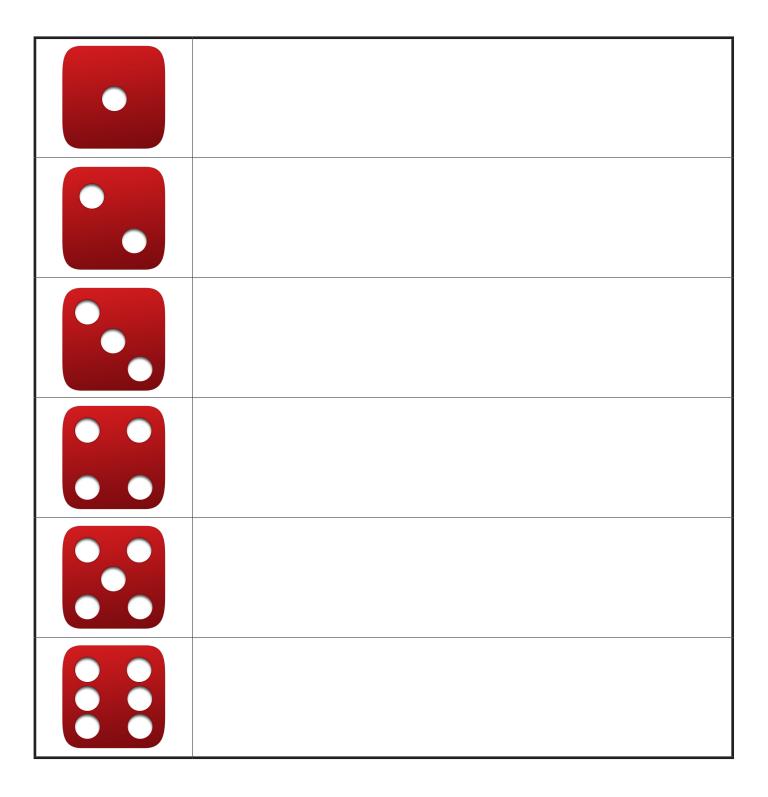
Instructions

- 1. Organize students into small groups.
- 2. Provide each group the following:
 - Rock and Roll Item Review template
 - Assessment item
 - Number cube
- 3. Give each student a Rock and Roll Item Review recording sheet.
- 4. Students take turns rolling the number cube and leading group members in analyzing the assessment item based on the task associated with the number rolled. All group members record the group's responses on their own template.
- 5. Students continue taking turns until they have completed all the components of the Rock and Roll Item Review template. Students who roll a number already completed lose a turn and the next group member takes a turn.
- 6. Upon completion of activity, students engage in Musical Mix and Freeze (or other instructional strategy from the Movement and Discourse list) to share responses.
- 7. Students return to their original group.
- 8. Model the problem-solving process to show students the solution to the problem.

Materials

- Assessment item
- Number cubes
- Rock and Roll Item Review Template/Recording Sheet (or project the prompts/questions and have students record responses on notebook paper/journals)

Rock and Roll Item Review – Recording Sheet



Rock and Roll Item Review – Template

	What are three important details about the problem?	What is confusing about the problem?
What does the problem want you to answer?		What model/strategy could be used to help with finding the solution?
	What information is not important to the problem?	Will this be a one-step or multi-step problem?
	What are three important details about the problem?	What is confusing about the problem?
	What does the problem want you to answer?	What model/strategy could be used to help with finding the solution?
	What information is not important to the problem?	Will this be a one-step or multi-step problem?

Toss a Question

Purpose

analyze information, formulate a plan/strategy, estimate

Instructions

- 1. Place students in groups of three. Ask students to number off #1 #3.
- 2. Provide each group with the same assessment question and a Toss a Question template.
- 3. In round one of the paper toss, group member #1 leads the task. Students work collaboratively to write down three known facts about the problem.
- 4. Upon completion of the task, all #1 students wad up their Toss a Question paper into a ball. They throw the paper ball three times, pick up the fourth paper ball, and bring it back to the group.
- 5. In round two of the paper toss, group member #2 leads the next task. Students work collaboratively to determine how to best represent the information from the problem using a drawing/diagram/image.
- 6. Upon completion of the task, all #2 students wad up their paper into a ball. They throw the paper ball three times, pick up the fourth paper ball, and bring it back to the group.
- 7. Students continue taking turns leading the paper toss until all of the identified attributes (e.g., three known facts; diagram/image; strip diagram/proportion; equation; estimation; solution) have been represented.
- 8. After final paper toss, students review the different representations for accuracy. Students should be prepared to share out corrections and/or enhancements that are needed for accuracy or clarification purposes.

Materials

- Assessment item
- Toss a Question template

Toss a Question – Elementary Template

Example: To make posters, 6 students each collected 8 pictures of animals. The students put 4 animals on each poster they made. How many posters did the students make?

3		
Whole Part Part	1+1=2	

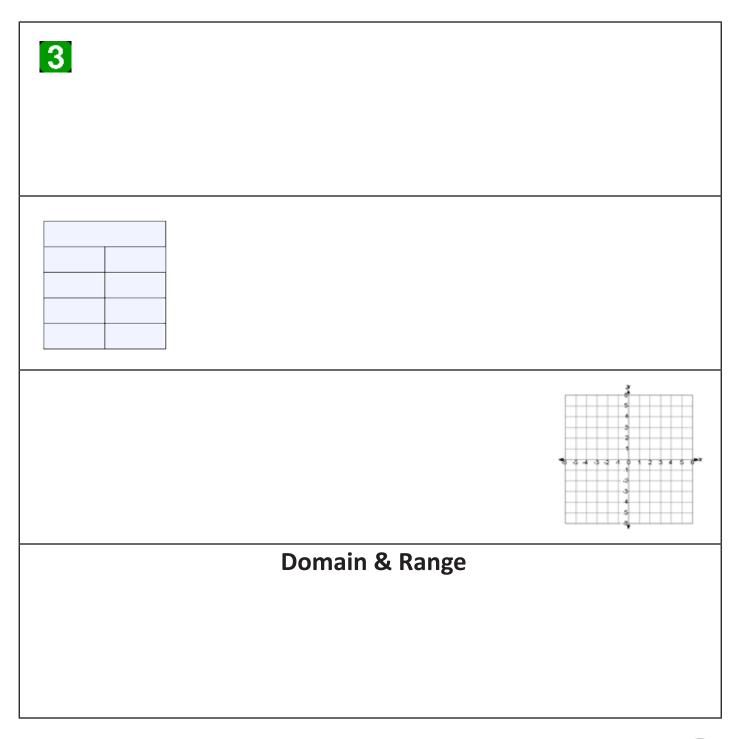
Toss a Question – Middle School Template

Example: A barrel contained 60 gallons of water. Water leaked out of the barrel at a rate of 5 gallons every 3 days. At this rate, how many days did it take for all 60 gallons of water to leak out of the barrel?

3	
Image: Constraint of the second of the se	
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Toss a Question – High School Template

Example: A set of weights includes a 4 lb barbell and 6 pairs of weight plates. Each pair of plates weighs 20 lb. If x pairs of plates are added to the barbell, the total weight of the barbell and plates in pounds can be represented by f(x) = 20x + 4.



Comprehension Tower

Purpose

communicate understanding, reasonableness, and conjectures

Instructions

- 1. Organize students into small groups.
- 2. Provide each group the following:
 - Set of blocks marked with four different colors
 - Comprehension Tower template
 - Set of math problems
 - Comprehension Tower recording sheet (optional)
- 3. Students participate in rock, paper, scissors to determine who in the group makes the first move. Moving blocks consists of taking one block from any level (except the one below the incomplete top level) of the tower. Only one hand should be used at a time when taking blocks from the tower. Blocks may be bumped to find a loose block that will not disturb the rest of the tower. Any block that is moved out of place must be returned to its original location before attempting to remove another block.
- 4. Students take turns removing a block, selecting a math problem from the pile of questions, and answering the associated color-coded question from the Comprehension Tower template. Teacher may elect to have students record their responses on the Comprehension Tower recording sheet.
- 5. Students return the question cards to the bottom of the stack but keep the blocks they removed from the tower.
- 6. The activity ends when the tower falls. Points may be assigned to each color-coded block (e.g., blue = 95 / 2.3 / -8; green = 150 / 1.5 / -5; red = 75 / ³/₄ / 7; purple = 85 / 2¹/₂ / -4). Students with the most points win the activity.

Materials

- Set of blocks marked with four different colors on the end
- Comprehension Tower Ways to Show template (color correlated to blocks)
- Set of math problems
- Comprehension Tower recording sheet (optional)

Comprehension Tower Color-Correlated Blocks and Ways to Show Template



Explain how to solve the problem

Explain how the solution is reasonable

Change a value in the problem and explain how that changes the solution

Add another step to the problem and explain how that changes the solution

Comprehension Tower – Sample Math Problems

 #1 A water dispenser contains 512 fluid ounces of water. What is the total number of 8-fluid-ounce cups of water that can be filled from the dispenser? Answer: 64 cups of water 	 #2 Sabra read a total of 185 pages in three days. One the first day, she read 85 pages On the second and third days, she read the same number of pages How many pages did she read on the third day?
#3 Mark had 45 football cards. Josh had twice as many football cards as Mark. Josh then bought 5 more football cards. How many football cards does Josh have now? <i>Answer</i> : 95 football cards	#4 A factory makes 400 refrigerators every day. The factory makes 125 more stoves per day than refrigerators. What is the total number of refrigerators and stoves the factory makes per day? <i>Answer</i> : 925 refrigerators
#5 Diane worked 18 hours each week during the summer. She worked a total of 8 weeks and earned \$9 an hour. How much money did Diane earn during the summer? <i>Answer</i> : \$1,296	#6 Terrell spent \$306 on a television and 3 video games. He spent \$243 on the television. Each video game was the same price. How much did Terrell spend on each video game? <i>Answer</i> : \$21

math

instructional strategies playlist – process standards

Comprehension Tower – Recording Sheet

add another step how does that change the solution?						
change a value how does that change the solution?						
explain how the solution is reasonable						
explain how to solve the problem						
Question Number	\leftarrow	5	S	4	Ь	9

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Ditto Dance Off

Purpose

communicate understanding and analyze connections

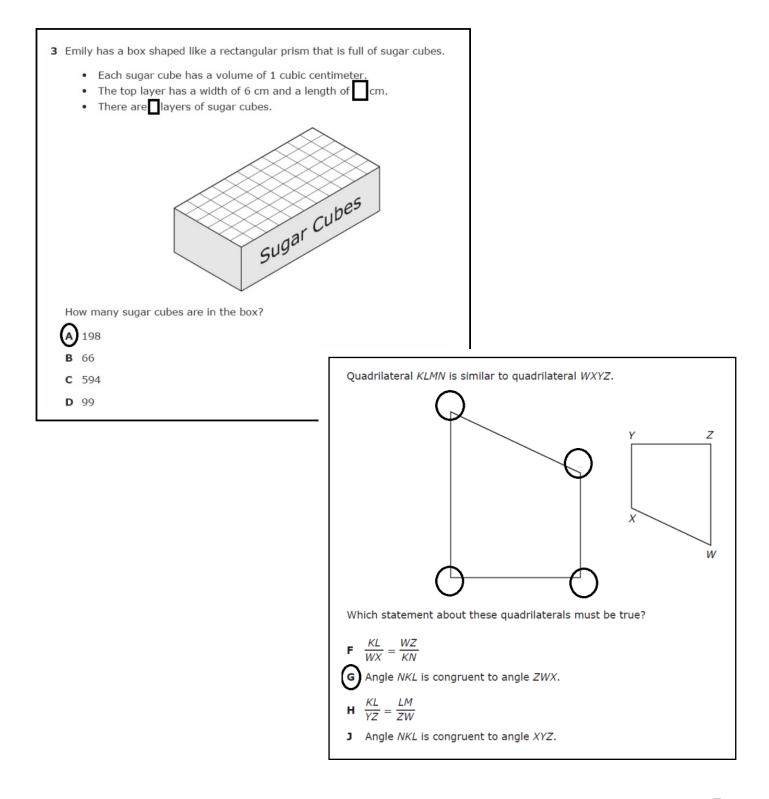
Instructions

- 1. Organize students into small groups.
- 2. Provide each student with an assessment item that identifies the correct answer and has missing components of the problem.
- 3. With their assigned assessment item, student groups work backward to determine the missing components of the problem.
- 4. Upon completion of the task, students partner up with someone from another group to engage in a Ditto Dance Off.
- 5. While the teacher plays music for 30 seconds, the taller student of the pair dances first, and the shorter student attempts to replicate their steps.
- 6. When the music stops, the taller student shares the "moves" they took to find the first missing component of the problem. The shorter student replicates/summarizes their understanding of what they just heard.
- 7. Repeat steps 5 and 6, allowing the shorter student to take the lead and unveil the second missing component of the problem.
- 8. Students return to their original group.
- 9. Clarify/verify the missing components of the problem with the whole class.

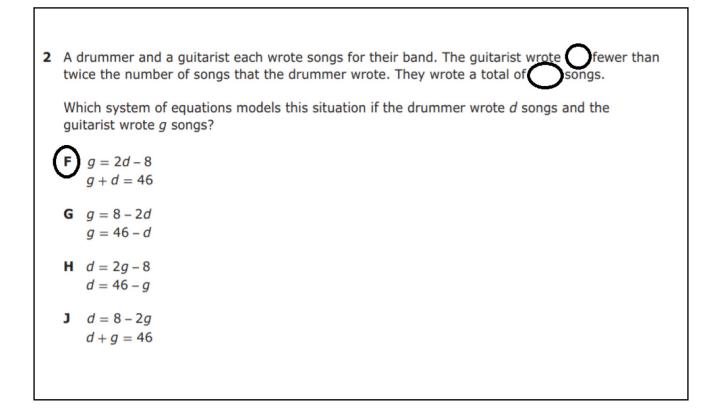
Materials

- Assessment item with correct answer and missing components of the problem
- Music
- Space to dance

Ditto Dance Off – Elementary/Middle School Examples



Ditto Dance Off – High School Example





Graffiti Art

Purpose

communicate understanding using multiple representations and analyze connections

Instructions

- 1. Organize students into groups.
- 2. Give each group a different colored marker.
- 3. Groups rotate through stations to observe a given assessment stimuli/image (e.g., graph, table, equation, geometric shape, clock, thermometer, etc.).
- 4. Groups select one of the images from the list of stimuli to represent the identified assessment item and mark off the stimuli/image used from the list.
- 5. When prompted, groups rotate to the next station, observe the different representation(s) created by previous group(s), and select a different way of representing the assessment item (representations may not be repeated).
- 6. Groups continue to rotate through the remaining stations until they return to their original station.
- 7. Students analyze the various representations for accuracy and prepare to share each of them with the class.

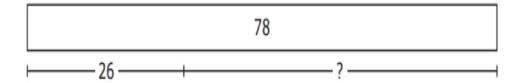
Materials

- Assessment items
- List of Stimuli/Images
- Poster/chart paper
- Markers

Think It Up

Have groups rotate through each station again, recording different real-world problems that could have been asked through the lens of the given stimuli.

Stimulus (visuals)
word problem
verbal description
chart/table
graph
number sentence/equation
diagram/image
number line
base ten blocks/algebra tiles
measurement tool
formula
geometric figure



Stimulus (visuals)
word problem
verbal description
chart/table
graph
number sentence/equation
diagram/image
number line
base ten blocks/algebra tiles
measurement tool
formula
geometric figure



Stimulus (visuals)
word problem
verbal description
chart/table
graph
number sentence/equation
diagram/image
number line
base ten blocks/algebra tiles
measurement tool
formula
geometric figure

Canoes and Paddleboats

Number of Canoes	Number of Paddleboats
72	90
37	55
61	79
85	103
68	86

Stimulus (visuals)
word problem
verbal description
chart/table
graph
number sentence/equation
diagram/image
number line
base ten blocks/algebra tiles
measurement tool
formula
geometric figure

The members of a gym use 98 towels every day. How many towels are used in 7 days?

Stimulus (visuals)
word problem
verbal description
chart/table
graph
number sentence/equation
diagram/image
number line
base ten blocks/algebra tiles
measurement tool
formula
geometric figure

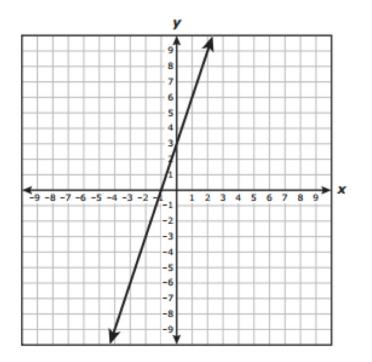
Caroline's Cell Phone Plan

Month	1	2	3	4
Total Amount Paid	\$32	\$64	\$96	\$128

Stimulus (visuals)
word problem
verbal description
chart/table
graph
number sentence/equation
diagram/image
number line
base ten blocks/algebra tiles
measurement tool
formula
geometric figure

Frank and his family drove 6 hours every day during a road trip.

Stimulus (visuals)
word problem
verbal description
chart/table
graph
number sentence/equation
diagram/image
number line
base ten blocks/algebra tiles
measurement tool
formula
geometric figure





Stimulus (visuals)
word problem
verbal description
chart/table
graph
number sentence/equation
diagram/image
number line
base ten blocks/algebra tiles
measurement tool
formula
geometric figure

y = 0.5x + 14

Justified List

Purpose

communicate understanding using multiple representations and analyze connections

Instructions

- 1. Organize students into small groups.
- 2. Give each student a Justified List recording sheet. The recording sheet includes a question and associated if/then statements to support students in demonstrating their "way of showing" how they understand the math problem.
- 3. Students read each statement, discuss their thinking, and record their justifications for each statement.
- 4. Students engage in a Learning Loop (see Movement and Discourse Playlist) to share their thoughts.
- 5. Teacher sees and hears students' thinking and clarifies/verifies as appropriate.

Materials

• Justified List recording sheet



Justified List Recording Sheet – Elementary Example

A factory makes 40 refrigerators every day. The factory makes 12 more stoves per day than refrigerators. How many stoves and refrigerators were made in three days?

- 40 refrigerators + 12 = 52 stoves
- 40 refrigerators x 3 days = 120 refrigerators
- 52 stoves x 3 days = 156 stoves
- 120 refrigerators + 156 stoves = 276 appliances made in three days

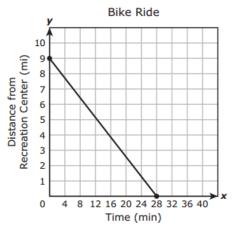
Read each statement and justify your response/thinking.

	Statements	Justification
1.	If the factory stopped producing stoves altogether, how would that impact the solution?	
2.	If the factory reduced the number of refrigerators it made by half, how would that impact the solution?	
3.	If the factory finds 2 defective refrigerators each day, how would that impact the solution?	
4.	How would the problem have to read if this strip diagram represented the problem?	
	Stoves Refrigerators	
	40 40 8	
5.	How would the problem have to read if this equation represented the problem?	
	50 + (50 — 18) = a	

Justified List Recording Sheet – Secondary Example

A student rode a bike from school to a recreation center. The graph shows the student's distance in miles from the recreation center after riding the bike for x minutes.

Read each statement and then justify your thinking.



Stat	ements	Justification
	axis went by increments of f 4 minutes, how would this	
	nter was 6 miles from school om school, how would this	
	ed to take a rest half way e, how would this impact	
4. If the student were would this impact th	riding his bike uphill, how ne graph?	
 If the student were riding the bike, how graph? 	driving a car instead of would this impact the	

Positive Pings

Purpose

communicate understanding using multiple representations and make connections

Instructions

- 1. Organize students into small groups.
- 2. Provide each group with a large plastic cup and a ping pong ball.
- 3. Taking turns, each student bounces a ping pong ball off the table in an attempt to make it in the cup. Give each student six attempts. Repeat this step for each group member.
- 4. Students answer questions about a math problem/assessment item based on their number of positive pings. For example:
 - 0-1 Explain the steps you took to solve the problem.
 - 2-3 Explain another way that you could solve the problem.
 - 4-5 Identify three math academic vocabulary words that relate to the math problem.
 - 6 Explain how this problem relates to another math concept or to something outside of math class.
- 5. As each group member shares their responses, other group members record the responses on their Positive Ping recording sheet or notebook paper/journal.

Materials

- Large plastic cups and ping pong balls
- Math problem/assessment item
- Positive Ping recording sheet (optional)

ways to show

Positive Pings – Recording Sheet

Explain the steps you took to solve the problem.	Explain another way that you could solve the problem.
Identify three math academic vocabulary words that relate to the math problem.	Explain how this problem relates to another math concept or to something outside of math class.

Rock and Roll Item Review

Purpose

communicate understanding using multiple representations, reasonableness, conjectures, and connections

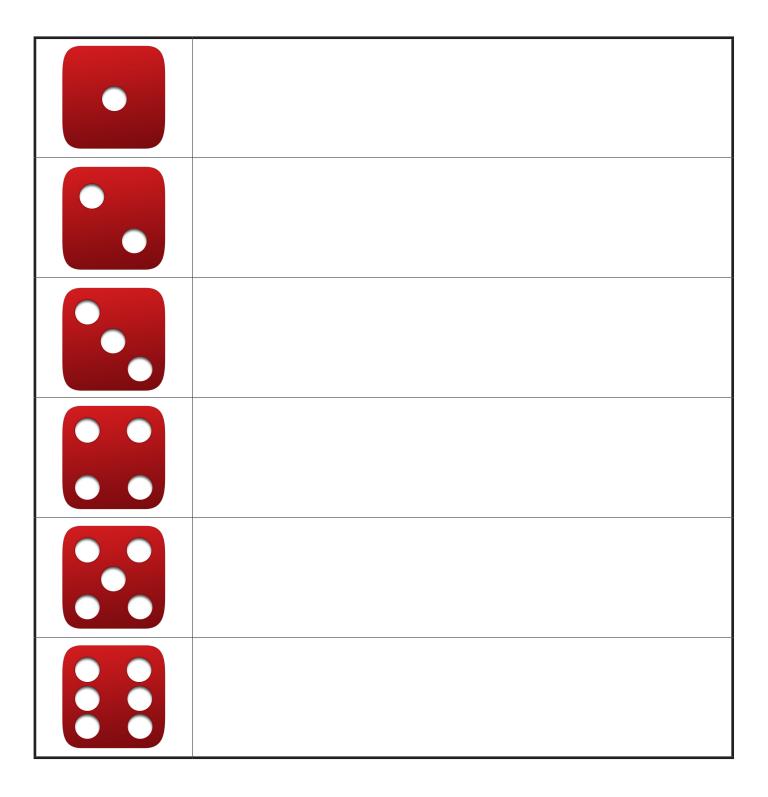
Instructions

- 1. Organize students into small groups.
- 2. Provide each group the following:
 - Rock and Roll Item Review template
 - Assessment item
 - Number cube
- 3. Give each student a Rock and Roll Item Review recording sheet.
- 4. Students take turns rolling the number cube and leading group members in analyzing the assessment item based on the task associated with the number rolled. All group members record the group's responses on their own template.
- 5. Students continue taking turns until they have completed all the components of the Rock and Roll Item Review template. Students who roll a number already completed lose a turn and the next group member takes a turn.
- 6. Upon completion of activity, students engage in Musical Mix and Freeze (or other instructional strategy from the Movement and Discourse list) to share responses.

Materials

- Assessment item
- Number cubes
- Rock and Roll Item Review Template/Recording Sheet (or project the prompts/questions and have students record responses on notebook paper/journals)

Rock and Roll Item Review – Recording Sheet



Rock and Roll Item Review – Template

	Explain the steps taken to solve the problem.	What other question could have been asked of the problem?
	Explain how the solution is reasonable.	Add another step to the problem and explain how that would change the solution.
	Represent the problem using a different stimuli/image.	Change a value in the problem and explain how that would change the solution.
~		
入		
	Explain the steps	What other question
	taken to solve the problem.	could have been asked of the problem?